**Number Theory I Homework Questions**

1. Given that 3 is a primitive root of 43, find
   (a) all positive integers less than 43 having order 6 modulo 43;
   (b) all positive integers less than 43 having order 21 modulo 43.

2. How many incongruent primitive does 50 have?

3. Suppose $r$ is a primitive root of 50. Find all incongruent primitive roots of 50 in terms of $r$.

4. Suppose $r$ is a primitive root of 50. Show that $r^3$ is also a primitive root of 50 and find all incongruent primitive roots of 50 in terms of $r^3$.

5. Suppose $r$ is a primitive root of 50. Find all incongruent integers having order 10.

6. How many positive integers $x$ are there such that $ord_{50}(x)=15$

7. Let $a \in \mathbb{Z}$ and suppose $r$ is a primitive root of 50 and $ind_r a = 8$. Then $ord_{50} a = ?$

8. Find all integers $x$ such that $100 \le x \le 500$ and $4 \mid x,\ 3 \mid x+1,\ 5 \mid x+3$

9. Find all integers $x$ such that $100 \le x \le 300, \quad 43!x = 1 \mod(51)$

10. Find the smallest positive integers $x$ such that $65!x = 5 \mod(71)$

11. Find the value of the Legendre symbol $\left(\dfrac{71}{101}\right)$

12. Is the quadratic congruence $x^2 \equiv 172 \mod(101)$ solvable?

13. Is 273 a quadratic residue of 101?

14. How many incongruent solutions of the linear congruence $6x \equiv 12 \mod(33)$ are there? Solve this congruence.

15. The following is a table of indices for the prime 17 relative to the primitive

root 3:

| $a$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $ind_3 a$ | 16 | 14 | 1 | 12 | 5 | 15 | 11 | 10 | 2 | 3 | 7 | 13 | 4 | 9 | 6 | 8 |

With the aid of this table, solve the congruences

(a) $x^{12} \equiv 13 \mod(17)$ (b) $8x^5 \equiv 10 \mod(17)$

(c) $9x^8 \equiv 8 \mod(17)$ (d) $7^x \equiv 7 \mod(17)$

16. Construct the table of indices for the prime 17 relative to the primitive

root 10.

17. Determine the integers $a$ $(1 \le a \le 12)$ such that the congruence

$ax^4 \equiv b \bmod(13)$ has a solution for $b = 2, 5,$ *and* $6.$

18. Decide whether or not the following quadratic conqruence equation are solvable. If solvable find the solutions.

(a) $x^2 + 5x + 13 \equiv 3 \bmod(11)$

(b) $3x^2 + 5x + 15 \equiv 4 \bmod(11)$

(c) $x^2 + 5x + 13 \equiv 3 \bmod(11)$

Reference: David M. Burton, Elementary Number Theory.