Chapter 3

Motion in Two or Three Dimensions

Learning Goals for Chapter 3

Looking forward at …

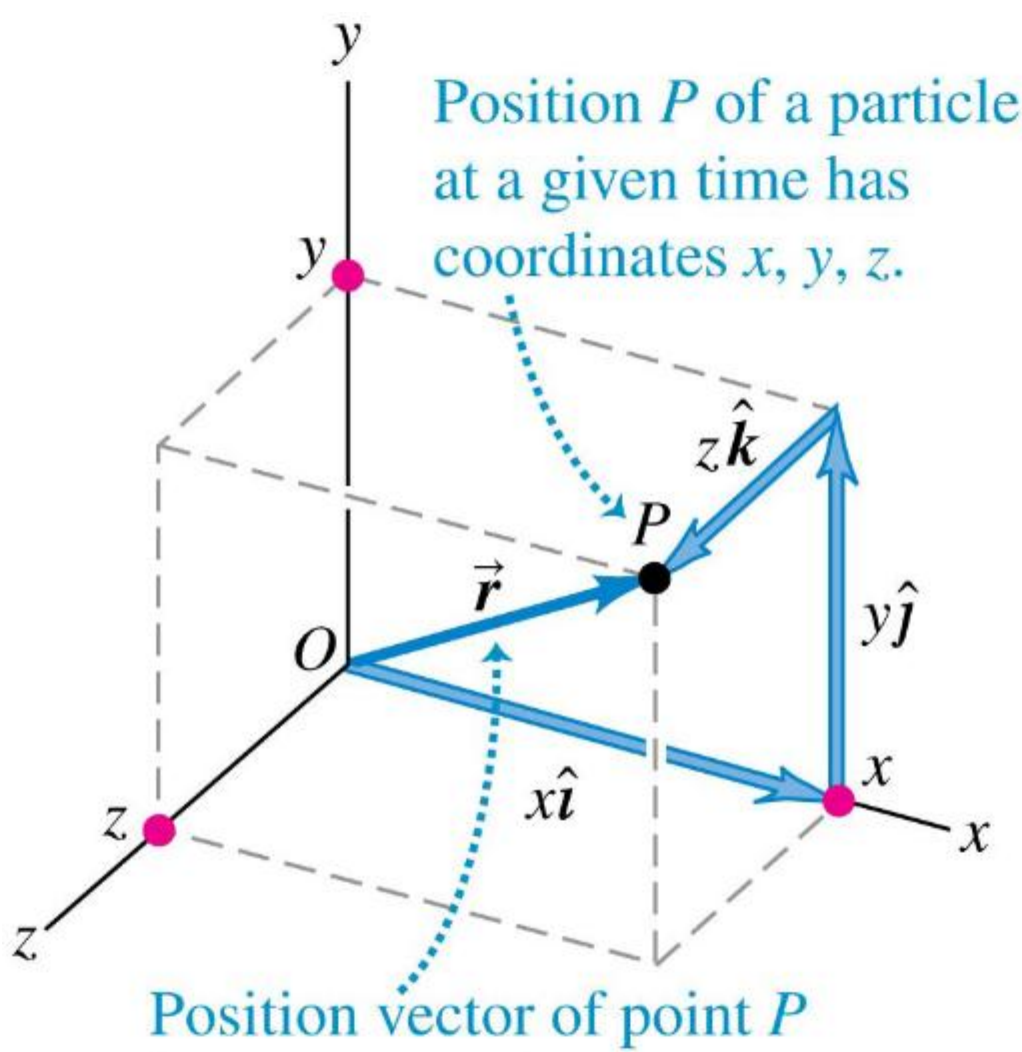
- how to use vectors to represent the position and velocity of a particle in two or three dimensions.
- how to find the vector acceleration of a particle, and how to interpret the components of acceleration parallel to and perpendicular to a particle's path.
- how to solve problems that involve the curved path followed by a projectile.
- how to analyze motion in a circular path, with either constant speed or varying speed.
- how to relate the velocities of a moving body as seen from two different frames of reference.

Introduction

- What determines where a batted baseball lands?
- How do you describe the motion of a roller coaster car along a curved track or the flight of a circling hawk?
- Which hits the ground first, a baseball that you simply drop or one that you throw horizontally?
- We need to extend our description of motion to two and three dimensions.

Position vector

- The position vector from the origin to point P has components x, y, and z.



Position vector of a particle at a given instant $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ Coordinates of particle's position

Unit vectors in x-, y-, and z-directions

Displacement

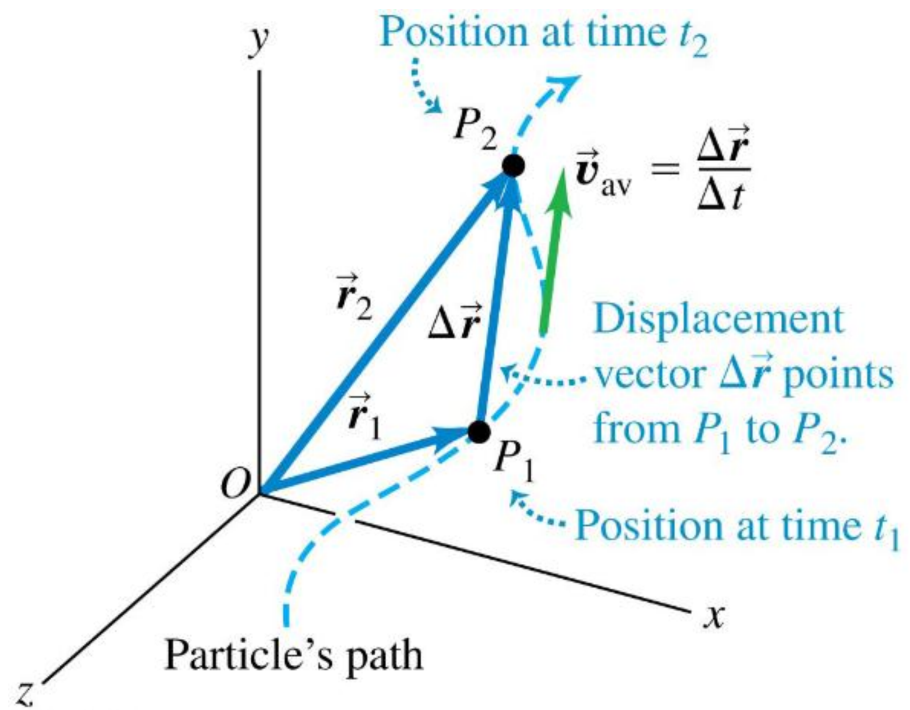
Position vector of a particle at a given instant

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

Coordinates of particle's position

Unit vectors in x-, y-, and z-directions

During a time interval Δt the particle moves from P_1, where its position vector is $\vec{r}_1$, to P_2, where its position vector is $\vec{r}_2$. The change in position (the displacement) during this interval is $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$.



Velocity

- The **average velocity** between two points is the displacement divided by the time interval between the two points, and it has the same direction as the displacement.

Change in the particle's position vector

Average velocity vector of a particle during time interval from t_1 to t_2

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

Final position minus initial position

Time interval

Final time minus initial time

- **Instantaneous velocity** (a.k.a. "velocity") is the instantaneous rate of change of position with time:

The **instantaneous velocity vector** of a particle ...

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

... equals the limit of its average velocity vector as the time interval approaches zero ...

... and equals the instantaneous rate of change of its position vector.

Instantaneous velocity

- The **instantaneous velocity** is the instantaneous rate of change of position vector with respect to time.

Each **component of** a particle's **instantaneous velocity vector** ...

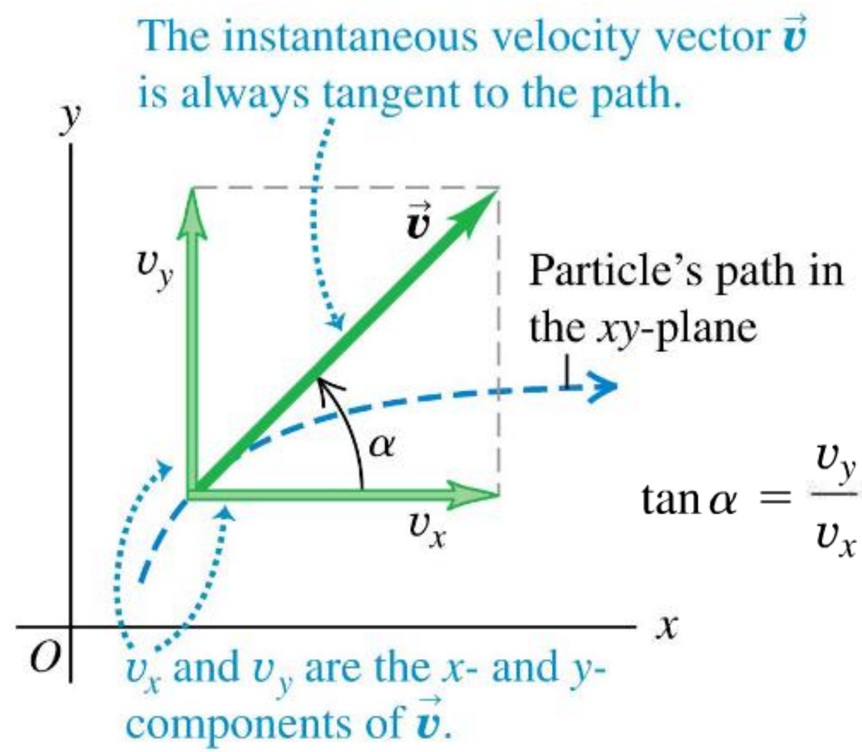
$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$

... equals the instantaneous rate of change of its corresponding coordinate.

- The instantaneous velocity of a particle is always tangent to its path.

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$

$$|\vec{v}| = v = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}$$



EXAMPLE 3.1 CALCULATING AVERAGE AND INSTANTANEOUS VELOCITY

A robotic vehicle, or rover, is exploring the surface of Mars. The stationary Mars lander is the origin of coordinates, and the surrounding Martian surface lies in the xy-plane. The rover, which we represent as a point, has x- and y-coordinates that vary with time:

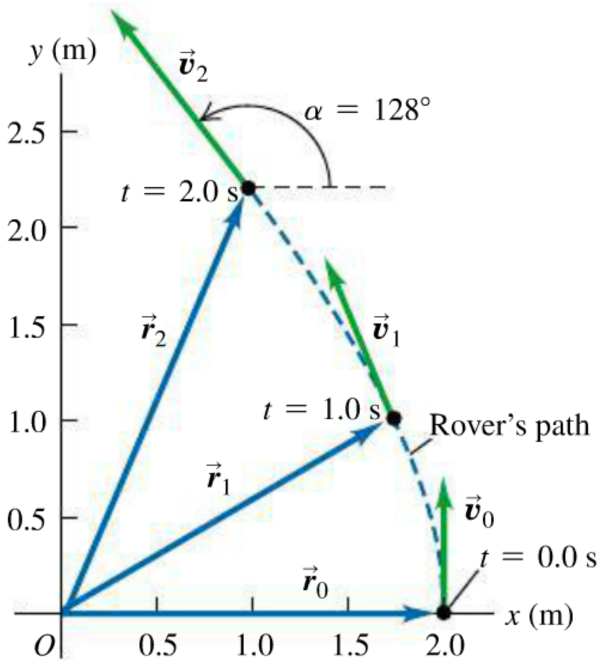
$$x = 2.0\ \text{m} - (0.25\ \text{m/s}^2)t^2$$
$$y = (1.0\ \text{m/s})t + (0.025\ \text{m/s}^3)t^3$$

(a) Find the rover's coordinates and distance from the lander at $t = 2.0$ s. (b) Find the rover's displacement and average velocity vectors for the interval $t = 0.0$ s to $t = 2.0$ s. (c) Find a general expression for the rover's instantaneous velocity vector $\vec{v}$. Express $\vec{v}$ at $t = 2.0$ s in component form and in terms of magnitude and direction.

EXAMPLE 3.1 CALCULATING AVERAGE AND INSTANTANEOUS VELOCITY

IDENTIFY and SET UP: This problem involves motion in two dimensions, so we must use the vector equations obtained in this section. **Figure 3.5** shows the rover's path (dashed line). We'll use Eq. (3.1) for position $\vec{r}$, the expression $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ for displacement, Eq. (3.2) for average velocity, and Eqs. (3.5), (3.6), and (3.7) for instantaneous velocity and its magnitude and direction.

3.5 At $t = 0.0$ s the rover has position vector $\vec{r}_0$ and instantaneous velocity vector $\vec{v}_0$. Likewise, $\vec{r}_1$ and $\vec{v}_1$ are the vectors at $t = 1.0$ s; $\vec{r}_2$ and $\vec{v}_2$ are the vectors at $t = 2.0$ s.



EXECUTE: (a) At $t = 2.0$ s the rover's coordinates are

$$x = 2.0\text{ m} - (0.25\text{ m/s}^2)(2.0\text{ s})^2 = 1.0\text{ m}$$

$$y = (1.0\text{ m/s})(2.0\text{ s}) + (0.025\text{ m/s}^3)(2.0\text{ s})^3 = 2.2\text{ m}$$

The rover's distance from the origin at this time is

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.0\text{ m})^2 + (2.2\text{ m})^2} = 2.4\text{ m}$$

(b) To find the displacement and average velocity over the given time interval, we first express the position vector $\vec{r}$ as a function of time t. From Eq. (3.1) this is

$$\begin{aligned}\vec{r} &= x\hat{\imath} + y\hat{\jmath}\\ &= [2.0\text{ m} - (0.25\text{ m/s}^2)t^2]\hat{\imath}\\ &\quad + [(1.0\text{ m/s})t + (0.025\text{ m/s}^3)t^3]\hat{\jmath}\end{aligned}$$

At $t = 0.0$ s the position vector $\vec{r}_0$ is

$$\vec{r}_0 = (2.0\text{ m})\hat{\imath} + (0.0\text{ m})\hat{\jmath}$$

From part (a), the position vector $\vec{r}_2$ at $t = 2.0$ s is

$$\vec{r}_2 = (1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath}$$

The displacement from $t = 0.0$ s to $t = 2.0$ s is therefore

$$\begin{aligned}\Delta\vec{r} = \vec{r}_2 - \vec{r}_0 &= (1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath} - (2.0\text{ m})\hat{\imath}\\ &= (-1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath}\end{aligned}$$

$$\begin{aligned}\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} &= \frac{(-1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath}}{2.0\text{ s} - 0.0\text{ s}}\\ &= (-0.50\text{ m/s})\hat{\imath} + (1.1\text{ m/s})\hat{\jmath}\end{aligned}$$

EXAMPLE 3.1 CALCULATING AVERAGE AND INSTANTANEOUS VELOCITY

(c) From Eq. (3.4) the components of *instantaneous* velocity are the time derivatives of the coordinates:

$$v_x = \frac{dx}{dt} = (-0.25\ \text{m/s}^2)(2t)$$

$$v_y = \frac{dy}{dt} = 1.0\ \text{m/s} + (0.025\ \text{m/s}^3)(3t^2)$$

Hence the instantaneous velocity vector is

$$\begin{aligned}\vec{\boldsymbol{v}} &= v_x\hat{\imath} + v_y\hat{\jmath}\\ &= (-0.50\ \text{m/s}^2)t\hat{\imath} + [1.0\ \text{m/s} + (0.075\ \text{m/s}^3)t^2]\,\hat{\jmath}\end{aligned}$$

At $t = 2.0$ s the velocity vector $\vec{\boldsymbol{v}}_2$ has components

$$v_{2x} = (-0.50\ \text{m/s}^2)(2.0\ \text{s}) = -1.0\ \text{m/s}$$

$$v_{2y} = 1\ .0\ \text{m/s} + (0.075\ \text{m/s}^3)(2.0\ \text{s})^2 = 1.3\ \text{m/s}$$

The magnitude of the instantaneous velocity (that is, the speed) at $t = 2.0$ s is

$$\begin{aligned}v_2 &= \sqrt{{v_{2x}}^2 + {v_{2y}}^2} = \sqrt{(-1.0\ \text{m/s})^2 + (1.3\ \text{m/s})^2}\\ &= 1.6\ \text{m/s}\end{aligned}$$

Figure 3.5 shows the direction of velocity vector $\vec{\boldsymbol{v}}_2$, which is at an angle α between 90° and 180° with respect to the positive x-axis. From Eq. (3.7) we have

$$\arctan\frac{v_y}{v_x} = \arctan\frac{1.3\ \text{m/s}}{-1.0\ \text{m/s}} = -52°$$

This is off by 180°; the correct value is $\alpha = 180° - 52° = 128°$, or 38° west of north.

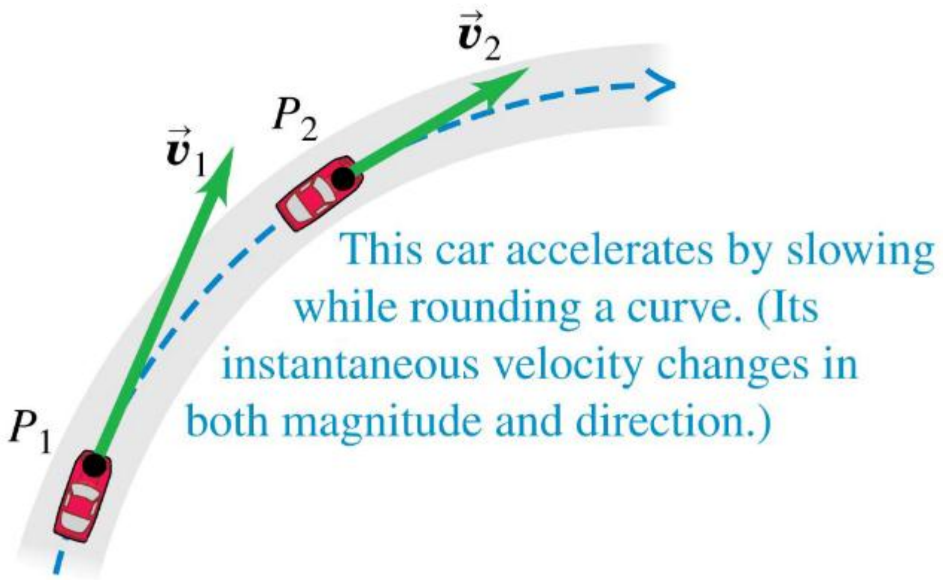
EVALUATE: Compare the components of *average* velocity from part (b) for the interval from $t = 0.0$ s to $t = 2.0$ s ($v_{\text{av-}x} = -0.50$ m/s, $v_{\text{av-}y} = 1.1$ m/s) with the components of *instantaneous* velocity at $t = 2.0$ s from part (c) ($v_{2x} = -1.0$ m/s, $v_{2y} = 1.3$ m/s). Just as in one dimension, the average velocity vector $\vec{\boldsymbol{v}}_{\text{av}}$ over an interval is in general *not* equal to the instantaneous velocity $\vec{\boldsymbol{v}}$ at the end of the interval (see Example 2.1).

Figure 3.5 shows the position vectors $\vec{\boldsymbol{r}}$ and instantaneous velocity vectors $\vec{\boldsymbol{v}}$ at $t = 0.0$ s, 1.0 s, and 2.0 s. (Calculate these quantities for $t = 0.0$ s and $t = 1.0$ s.) Notice that $\vec{\boldsymbol{v}}$ is tangent to the path at every point. The magnitude of $\vec{\boldsymbol{v}}$ increases as the rover moves, which means that its speed is increasing.

Acceleration

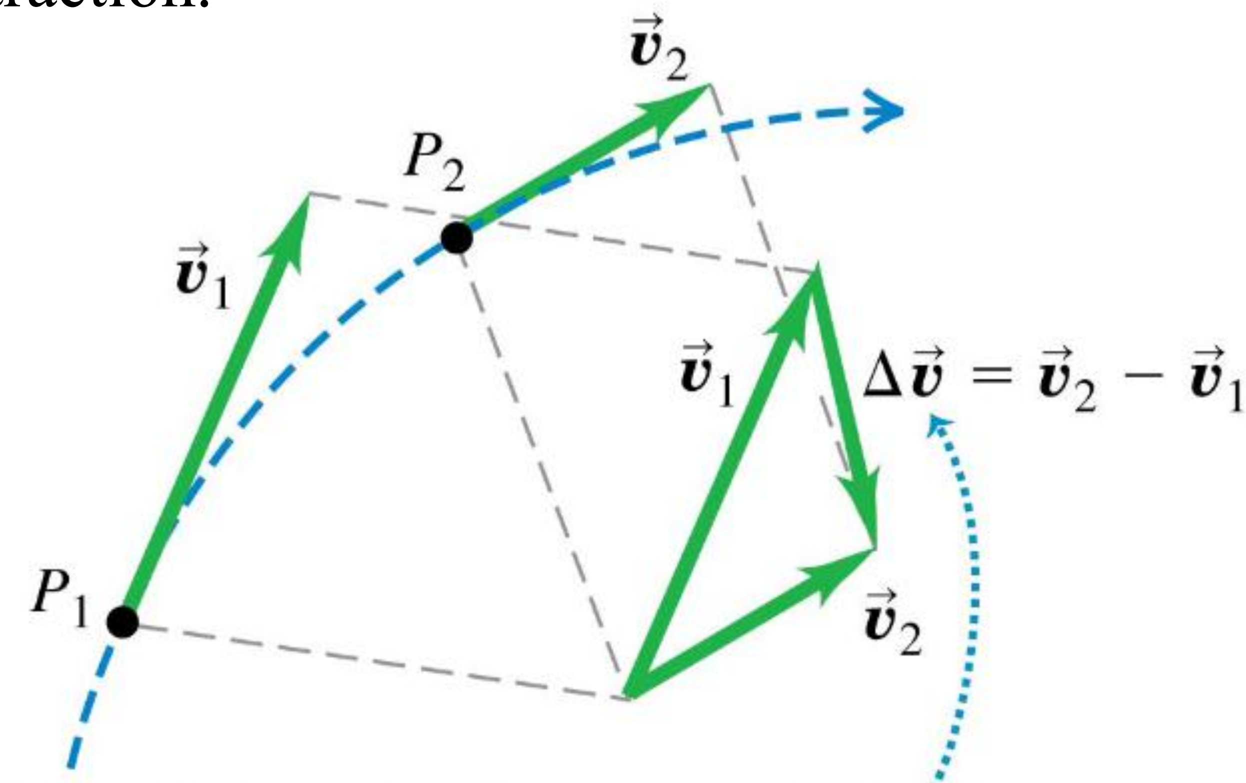
Now let's consider the *acceleration* of a particle moving in space. Just as for motion in a straight line, acceleration describes how the velocity of the particle changes. But since we now treat velocity as a vector, acceleration will describe changes in the velocity magnitude (that is, the speed) *and* changes in the direction of velocity (that is, the direction in which the particle is moving).

- Acceleration describes how the velocity changes.



Average acceleration

- The change in velocity between two points is determined by vector subtraction.



To find the car's average acceleration between P_1 and P_2, we first find the change in velocity $\Delta\vec{v}$ by subtracting $\vec{v}_1$ from $\vec{v}_2$. (Notice that $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$.)

Acceleration

- We define the **average acceleration** as the change in velocity divided by the time interval:

Change in the particle's velocity

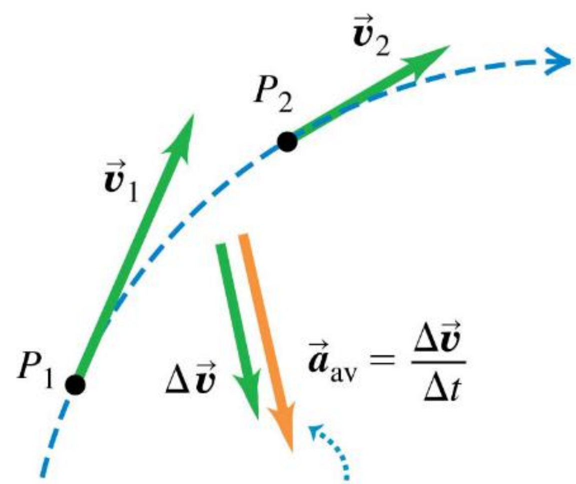
Average acceleration vector of a particle during time interval from t_1 to t_2

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Final velocity minus initial velocity

Time interval

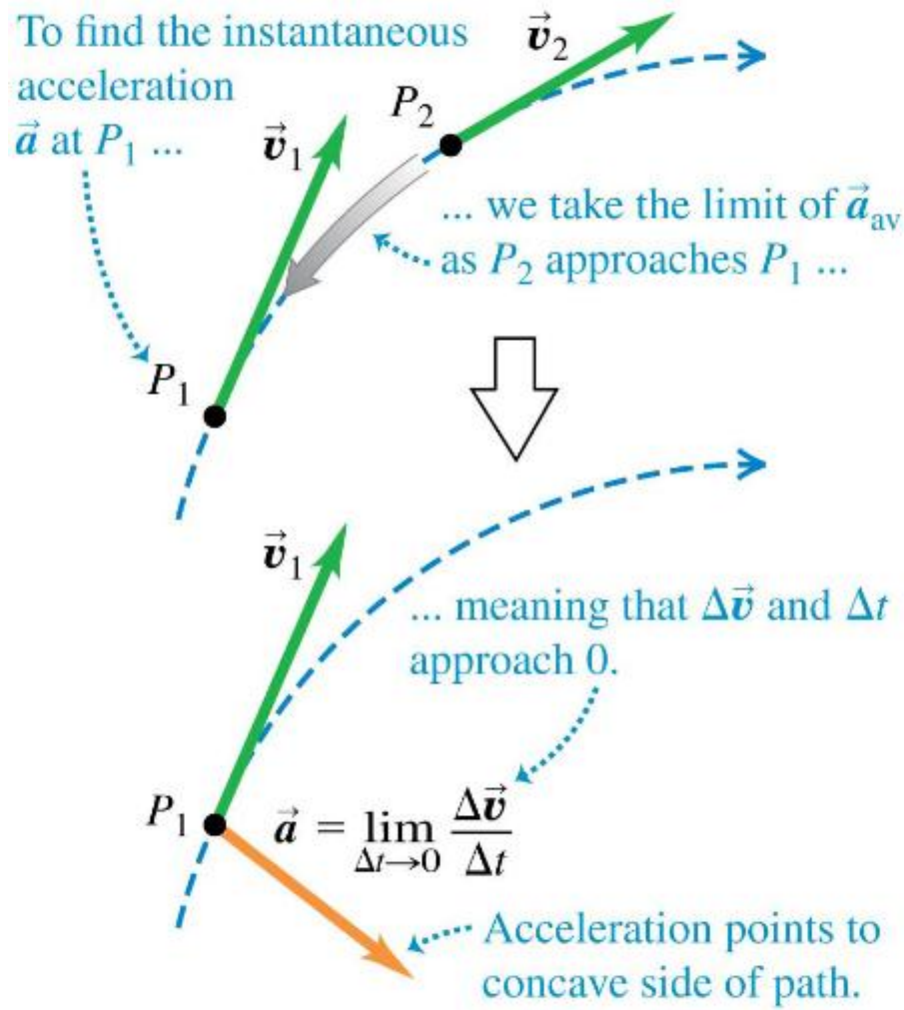
Final time minus initial time



The average acceleration has the same direction as the change in velocity, $\Delta\vec{v}$.

Instantaneous acceleration

- The velocity vector is always tangent to the particle's path, but the instantaneous acceleration vector does *not* have to be tangent to the path.
- If the path is curved, the acceleration points toward the concave side of the path.



Acceleration

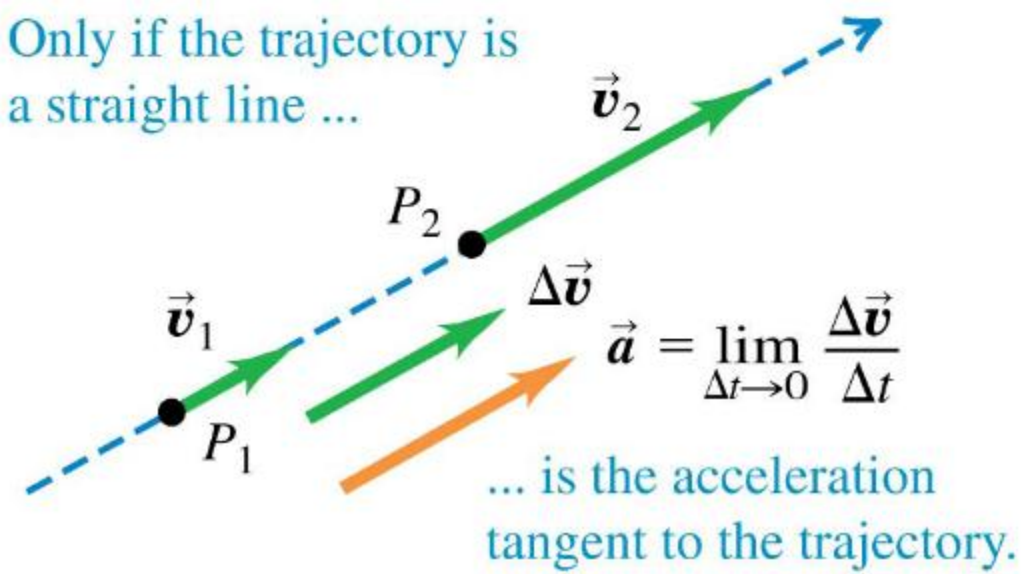
- **Instantaneous acceleration** (a.k.a. "acceleration") is the instantaneous rate of change of velocity with time:

The **instantaneous acceleration vector** of a particle ...

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

... equals the limit of its average acceleration vector as the time interval approaches zero ...

... and equals the instantaneous rate of change of its velocity vector.



Components of acceleration

Each **component of** a particle's **instantaneous acceleration vector** ...

$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt} \qquad a_z = \frac{dv_z}{dt}$$

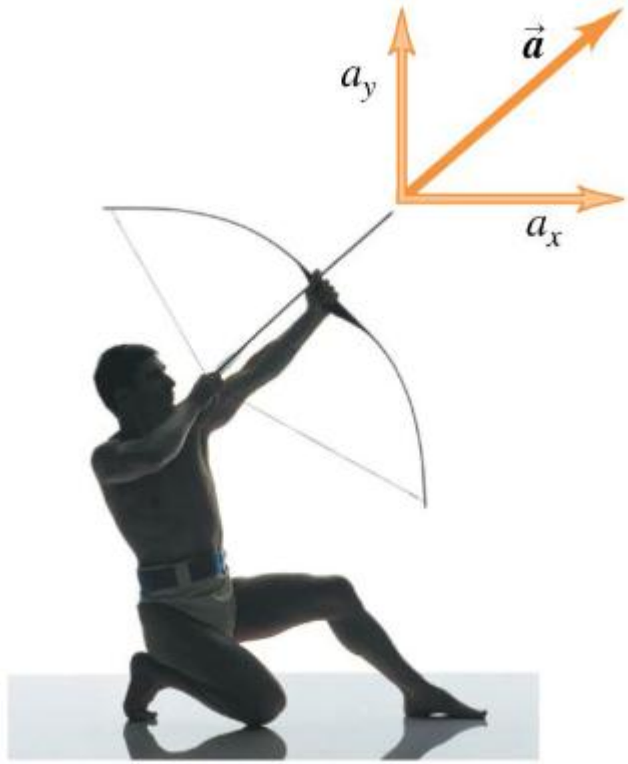
... equals the instantaneous rate of change of its corresponding velocity component.

- Shooting an arrow is an example of an acceleration vector that has both *x*- and *y*-components.

$$\vec{\boldsymbol{a}} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}$$

$$\vec{\boldsymbol{a}} = \frac{d\vec{\boldsymbol{v}}}{dt} = \frac{dv_x}{dt}\hat{\boldsymbol{\imath}} + \frac{dv_y}{dt}\hat{\boldsymbol{\jmath}} + \frac{dv_z}{dt}\hat{\boldsymbol{k}}$$

$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2} \qquad a_z = \frac{d^2z}{dt^2}$$



EXAMPLE 3.2 CALCULATING AVERAGE AND INSTANTANEOUS ACCELERATION

Let's return to the motions of the Mars rover in Example 3.1. (a) Find the components of the average acceleration for the interval $t = 0.0$ s to $t = 2.0$ s. (b) Find the instantaneous acceleration at $t = 2.0$ s.

SOLUTION

IDENTIFY and SET UP: In Example 3.1 we found the components of the rover's instantaneous velocity at any time t:

$$v_x = \frac{dx}{dt} = (-0.25\ \mathrm{m/s^2})(2t) = (-0.50\ \mathrm{m/s^2})t$$

$$v_y = \frac{dy}{dt} = 1.0\ \mathrm{m/s} + (0.025\ \mathrm{m/s^3})(3t^2)$$

$$= 1.0\ \mathrm{m/s} + (0.075\ \mathrm{m/s^3})t^2$$

EXECUTE: (a) In Example 3.1 we found that at $t = 0.0$ s the velocity components are

$$v_x = 0.0\ \mathrm{m/s} \qquad v_y = 1.0\ \mathrm{m/s}$$

and that at $t = 2.0$ s the components are

$$v_x = -1.0\ \mathrm{m/s} \qquad v_y = 1.3\ \mathrm{m/s}$$

Thus the components of average acceleration in the interval $t = 0.0$ s to $t = 2.0$ s are

$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{-1.0\ \mathrm{m/s} - 0.0\ \mathrm{m/s}}{2.0\ \mathrm{s} - 0.0\ \mathrm{s}} = -0.50\ \mathrm{m/s^2}$$

$$a_{\text{av-}y} = \frac{\Delta v_y}{\Delta t} = \frac{1.3\ \mathrm{m/s} - 1.0\ \mathrm{m/s}}{2.0\ \mathrm{s} - 0.0\ \mathrm{s}} = 0.15\ \mathrm{m/s^2}$$

(b) Using Eqs. (3.10), we find

$$a_x = \frac{dv_x}{dt} = -0.50\ \mathrm{m/s^2} \qquad a_y = \frac{dv_y}{dt} = (0.075\ \mathrm{m/s^3})(2t)$$

EXAMPLE 3.2 CALCULATING AVERAGE AND INSTANTANEOUS ACCELERATION

Hence the instantaneous acceleration vector $\vec{a}$ at time t is

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = (-0.50\ \text{m/s}^2)\hat{\imath} + (0.15\ \text{m/s}^3)t\hat{\jmath}$$

At $t = 2.0$ s the components of acceleration and the acceleration vector are

$$a_x = -0.50\ \text{m/s}^2 \qquad a_y = (0.15\ \text{m/s}^3)(2.0\ \text{s}) = 0.30\ \text{m/s}^2$$

$$\vec{a} = (-0.50\ \text{m/s}^2)\hat{\imath} + (0.30\ \text{m/s}^2)\hat{\jmath}$$

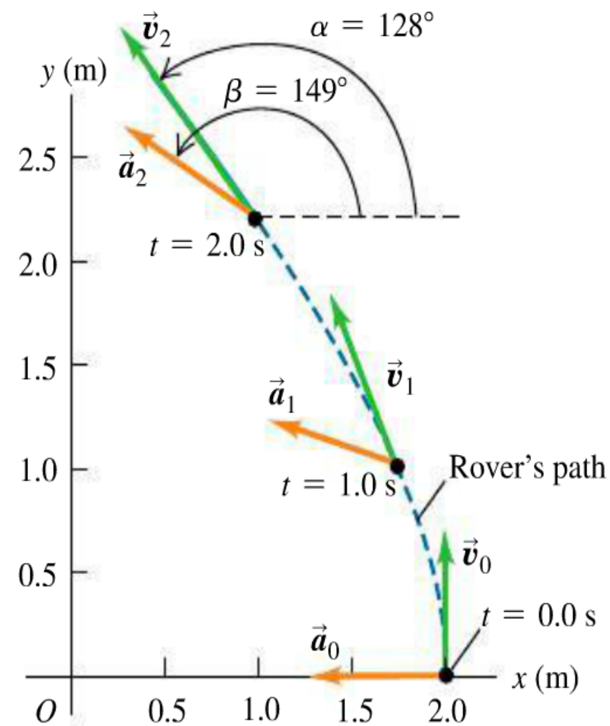
The magnitude of acceleration at this time is

$$a = \sqrt{a_x^2 + a_y^2}$$
$$= \sqrt{(-0.50\ \text{m/s}^2)^2 + (0.30\ \text{m/s}^2)^2} = 0.58\ \text{m/s}^2$$

A sketch of this vector (**Fig. 3.9**) shows that the direction angle β of $\vec{a}$ with respect to the positive x-axis is between 90° and 180°. From Eq. (3.7) we have

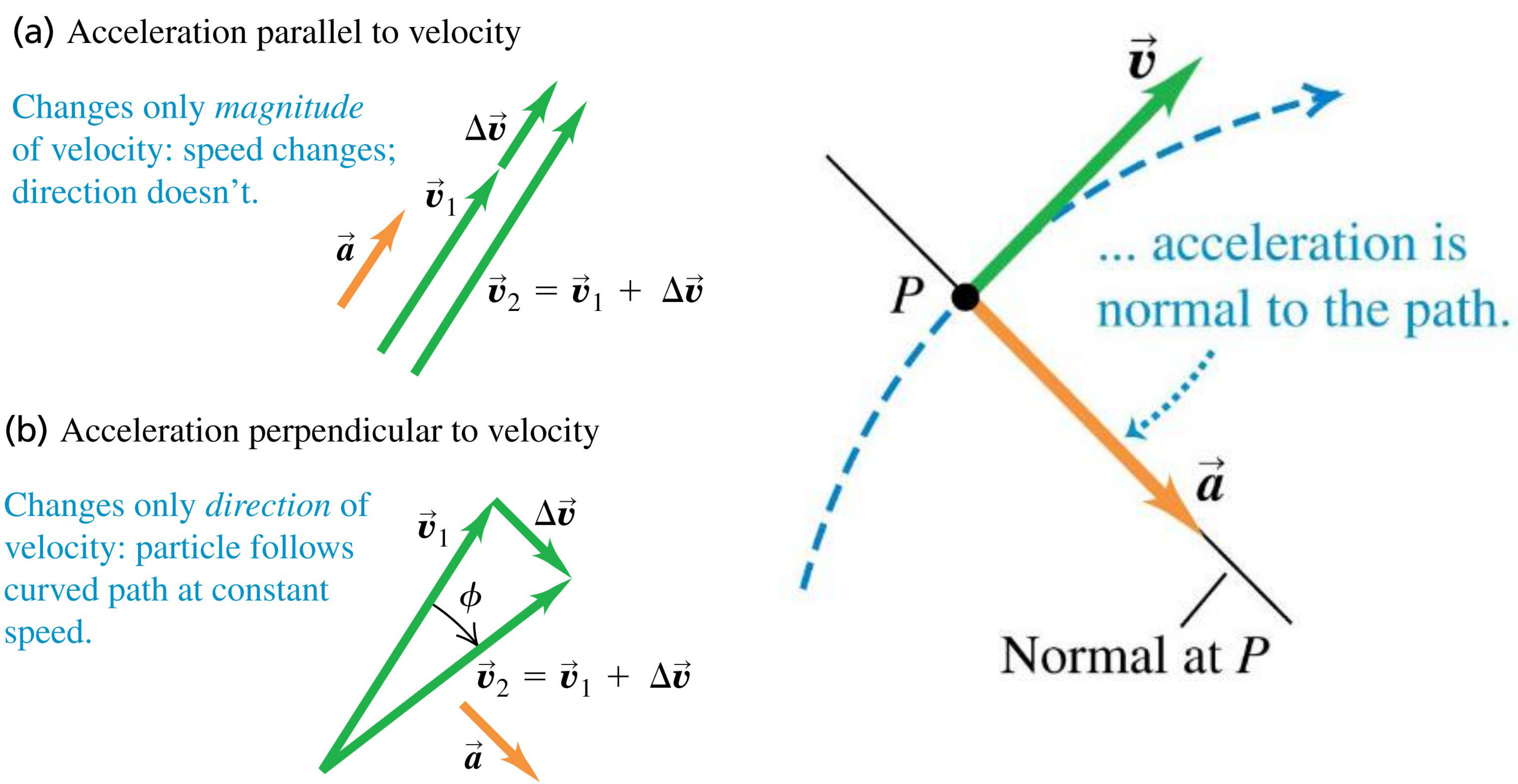
$$\arctan\frac{a_y}{a_x} = \arctan\frac{0.30\ \text{m/s}^2}{-0.50\ \text{m/s}^2} = -31°$$

Hence $\beta = 180° + (-31°) = 149°$.



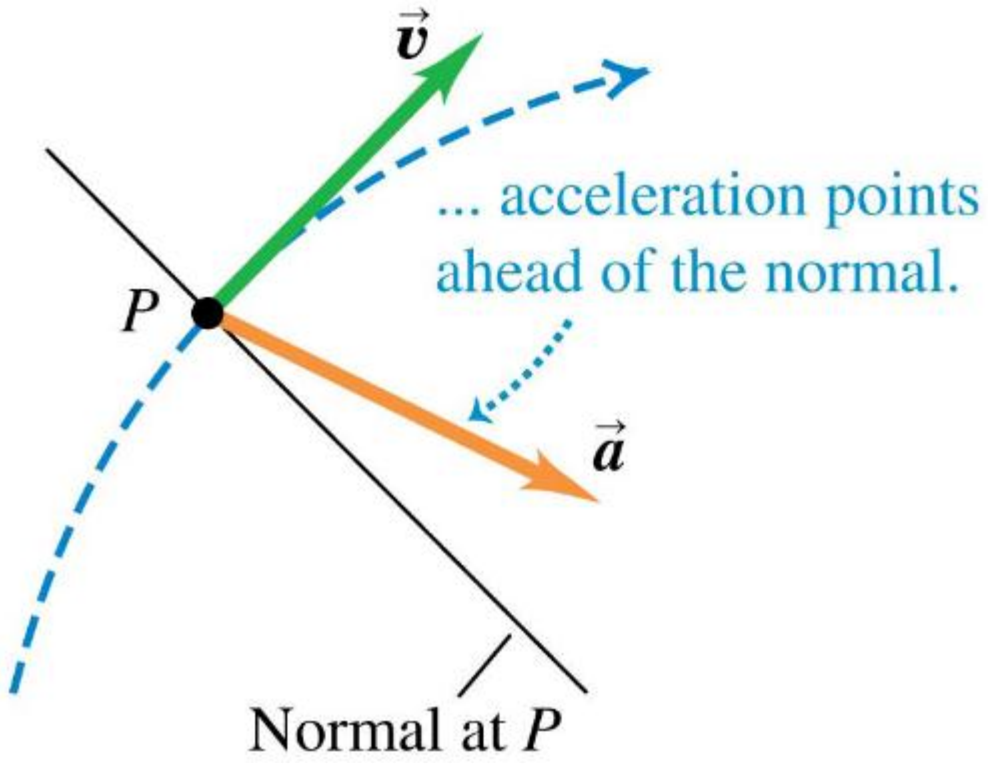
Parallel and perpendicular components of acceleration

- Velocity and acceleration vectors for a particle moving through a point *P* on a curved path with *constant speed*



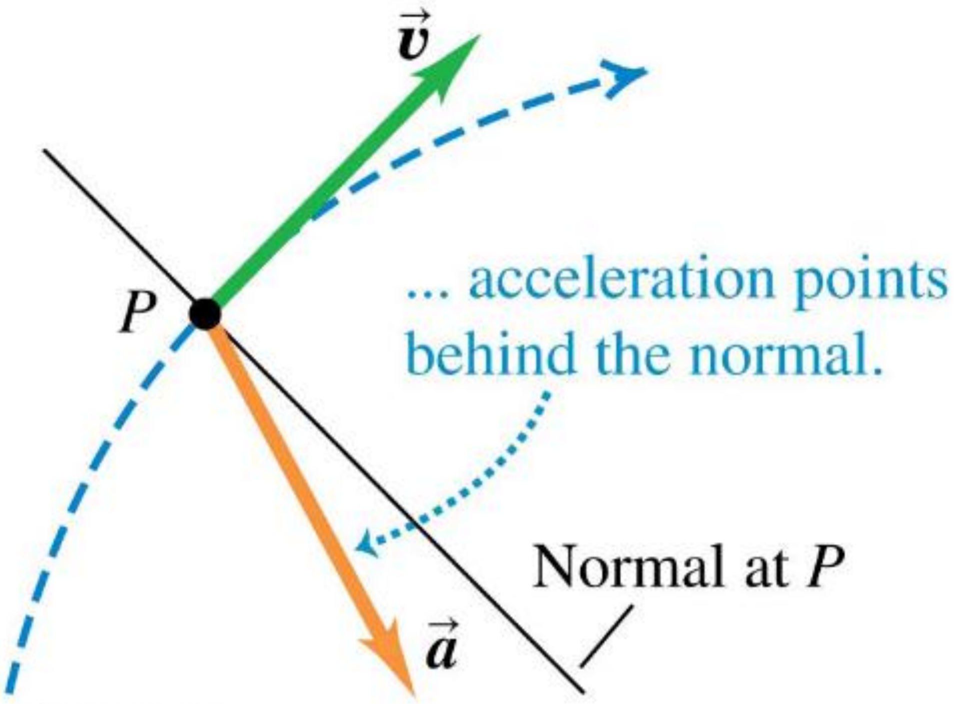
Parallel and perpendicular components of acceleration

- Velocity and acceleration vectors for a particle moving through a point P on a curved path with *increasing speed*



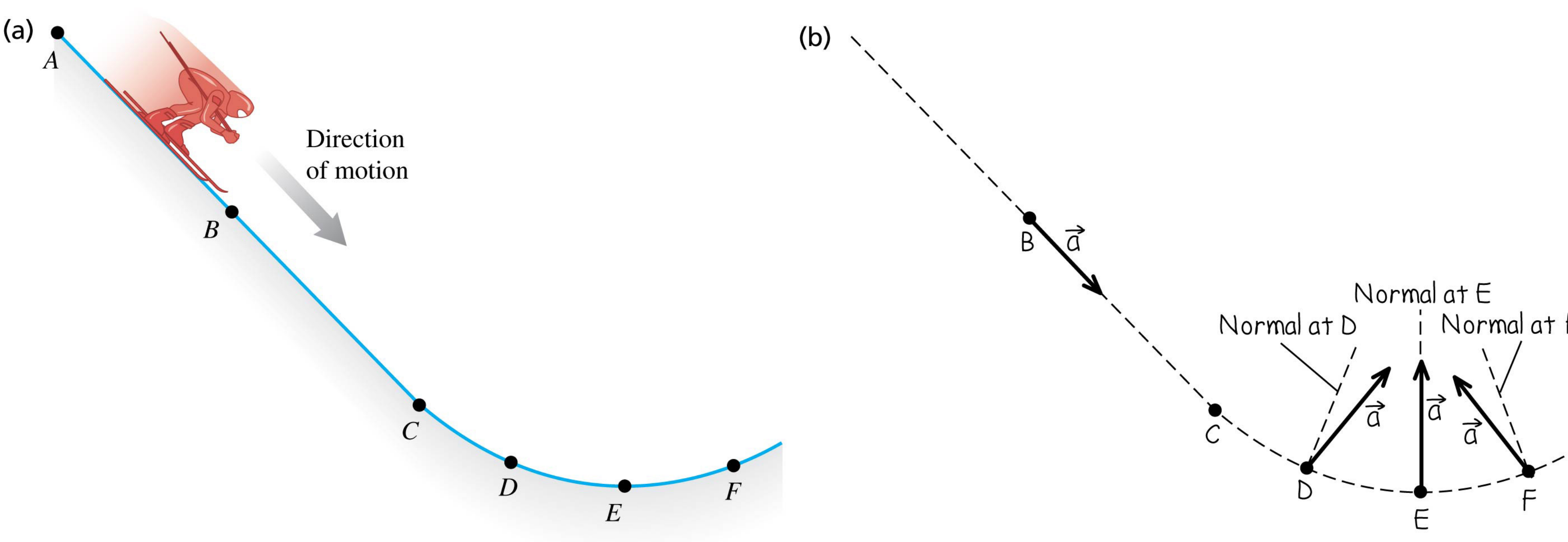
Parallel and perpendicular components of acceleration

- Velocity and acceleration vectors for a particle moving through a point P on a curved path with *decreasing speed*



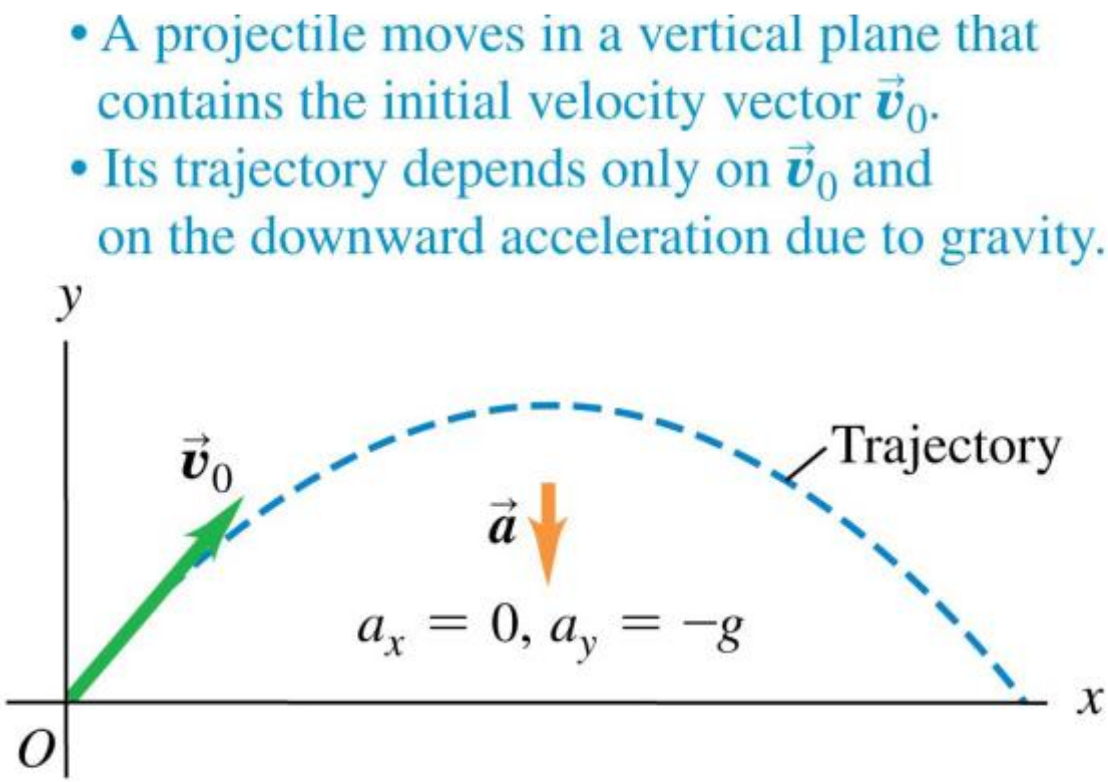
CONCEPTUAL EXAMPLE 3.4 ACCELERATION OF A SKIER

A skier moves along a ski-jump ramp (**Fig. 3.14a**). The ramp is straight from point *A* to point *C* and curved from point *C* onward. The skier speeds up as she moves downhill from point *A* to point *E*, where her speed is maximum. She slows down after passing point *E*. Draw the direction of the acceleration vector at each of the points *B*, *D*, *E*, and *F*.



Projectile motion

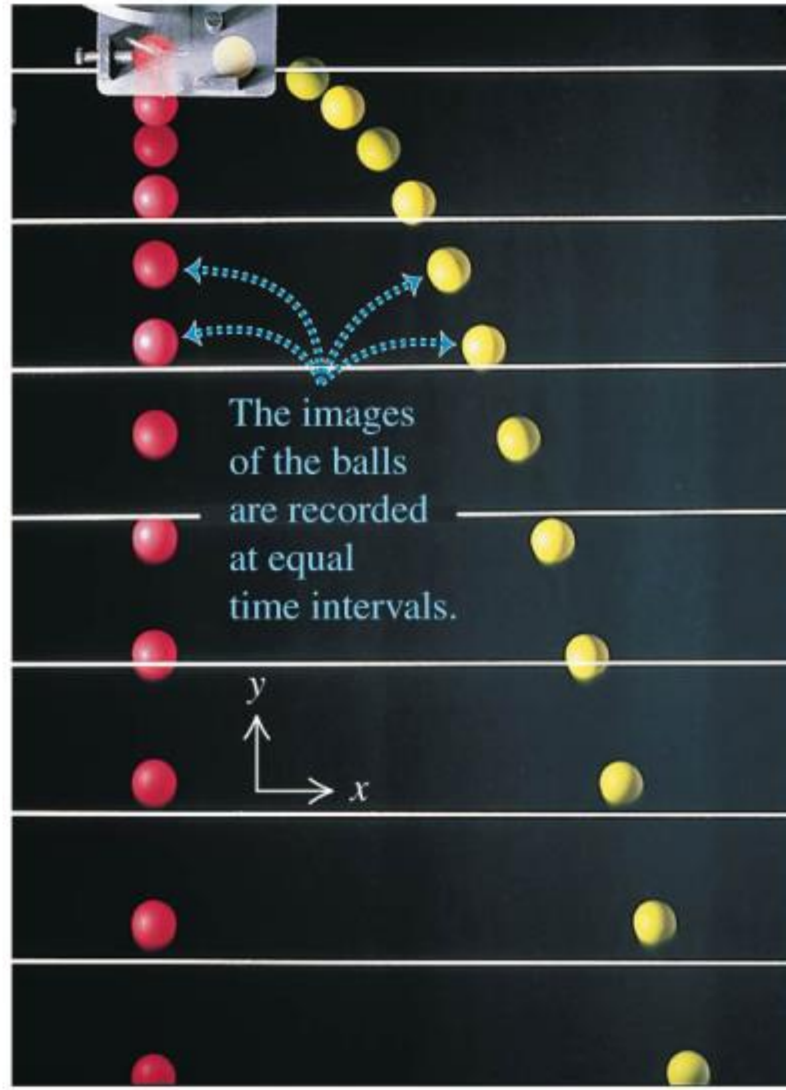
- A **projectile** is any body given an initial velocity that then follows a path determined by the effects of gravity and air resistance.
- Begin by neglecting resistance and the curvature and rotation of the earth.



The *x*- and *y*-motion are separable

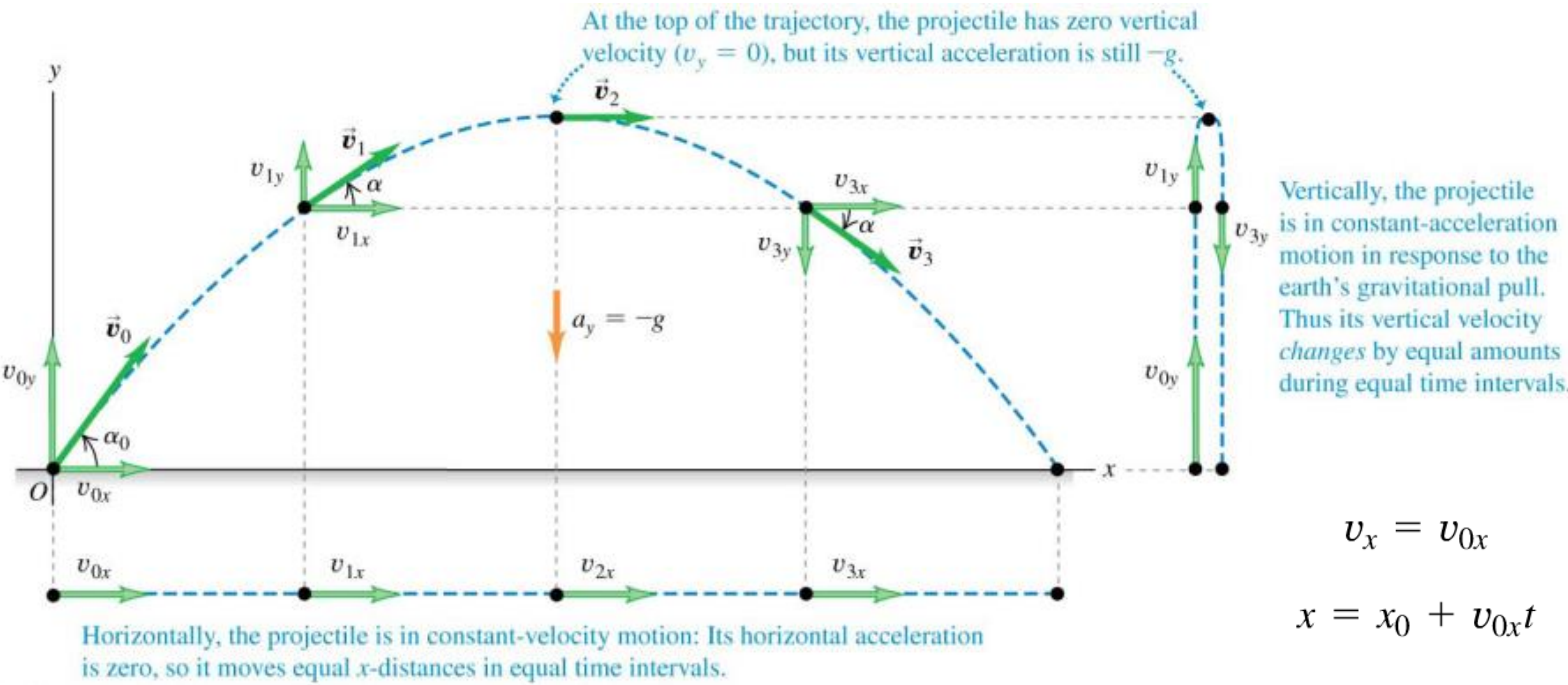
- The red ball is dropped at the same time that the yellow ball is fired horizontally.
- The strobe marks equal time intervals.
- We can analyze projectile motion as horizontal motion with constant velocity and vertical motion with constant acceleration:

  $a_x = 0$ and $a_y = -g$.



Projectile motion

- If air resistance is negligible, the trajectory of a projectile is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.



$$v_x = v_{0x}$$

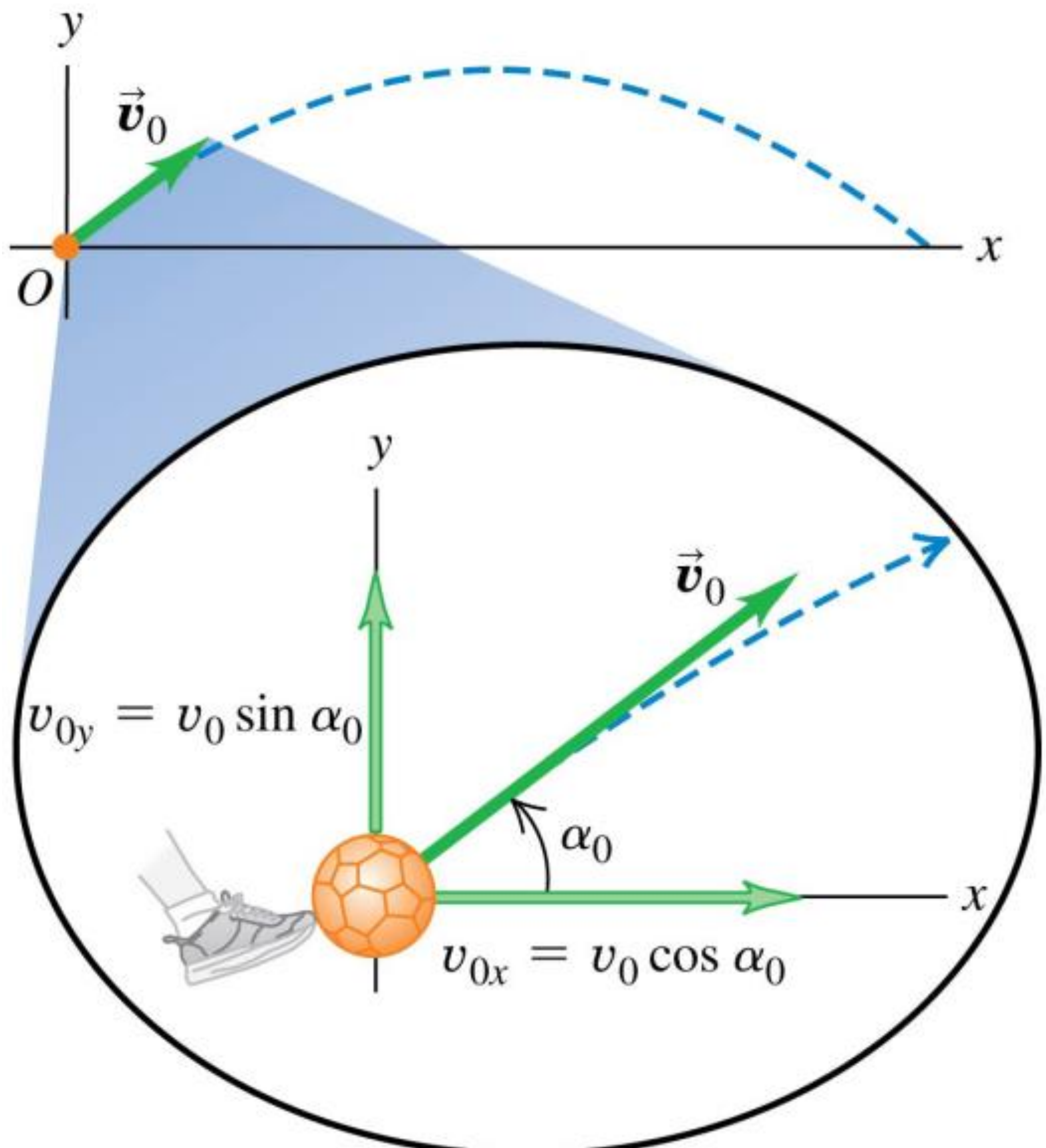
$$x = x_0 + v_{0x}t$$

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

Projectile motion – Initial velocity

- The initial velocity components of a projectile (such as a kicked soccer ball) are related to the initial speed and initial angle.



The equations for projectile motion

- If we set $x_0 = y_0 = 0$, the equations describing projectile motion are shown below:

Coordinates at time t of a **projectile** (positive y-direction is upward, and $x = y = 0$ at $t = 0$)

$$x = (v_0 \cos \alpha_0)t$$

$$y = (v_0 \sin \alpha_0)t - \tfrac{1}{2}gt^2$$

Speed at $t = 0$

Direction at $t = 0$

Time

Acceleration due to gravity: Note $g > 0$.

Velocity components at time t of a **projectile** (positive y-direction is upward)

$$v_x = v_0 \cos \alpha_0$$

$$v_y = v_0 \sin \alpha_0 - gt$$

Speed at $t = 0$

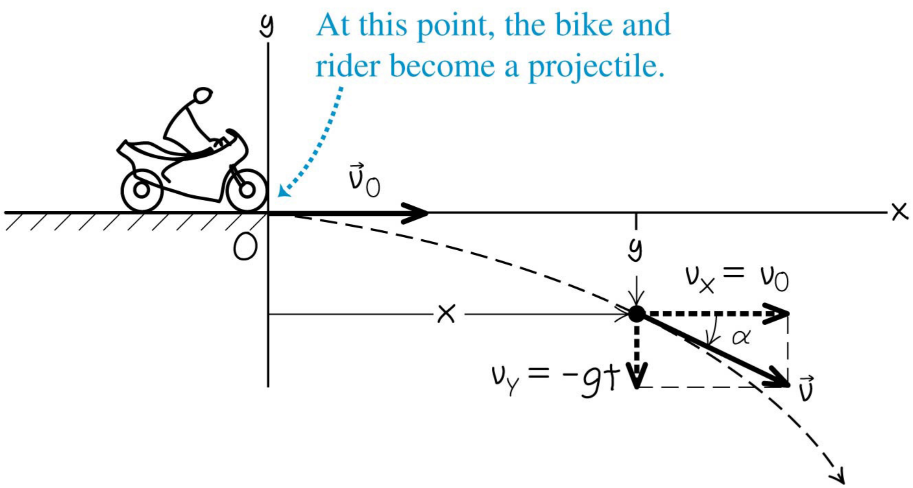
Direction at $t = 0$

Time

$$y = (\tan \alpha_0)x - \frac{g}{2v_0^2 \cos^2 \alpha_0}x^2$$

EXAMPLE 3.6 A BODY PROJECTED HORIZONTALLY

A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9.0 m/s. Find the motorcycle's position, distance from the edge of the cliff, and velocity 0.50 s after it leaves the edge of the cliff.



$$x = v_{0x}t = (9.0\ \text{m/s})(0.50\ \text{s}) = 4.5\ \text{m}$$

$$y = -\tfrac{1}{2}gt^2 = -\tfrac{1}{2}(9.80\ \text{m/s}^2)(0.50\ \text{s})^2 = -1.2\ \text{m}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(4.5\ \text{m})^2 + (-1.2\ \text{m})^2} = 4.7\ \text{m}$$

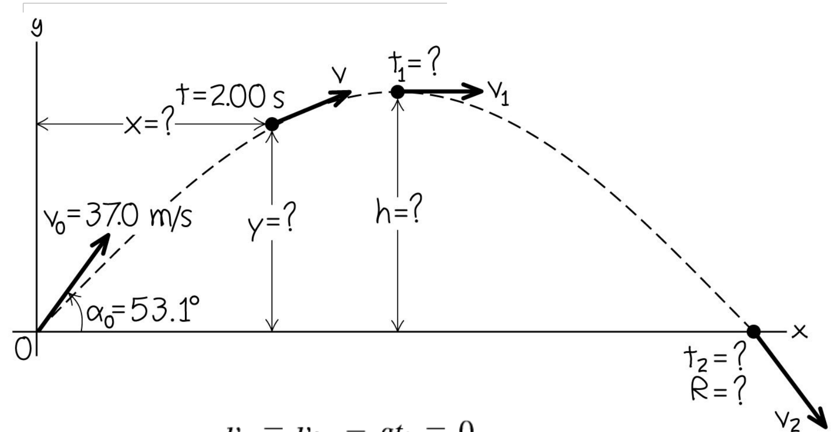
$$v_x = v_{0x} = 9.0\ \text{m/s}$$

$$v_y = -gt = (-9.80\ \text{m/s}^2)(0.50\ \text{s}) = -4.9\ \text{m/s}$$

$$\vec{\boldsymbol{v}} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} = (9.0\ \text{m/s})\hat{\boldsymbol{\imath}} + (-4.9\ \text{m/s})\hat{\boldsymbol{\jmath}}$$

EXAMPLE 3.7 HEIGHT AND RANGE OF A PROJECTILE I: A BATTED BASEBALL

A batter hits a baseball so that it leaves the bat at speed $v_0 = 37.0\text{ m/s}$ at an angle $\alpha_0 = 53.1°$. (a) Find the position of the ball and its velocity (magnitude and direction) at $t = 2.00\text{ s}$. (b) Find the time when the ball reaches the highest point of its flight, and its height h at this time. (c) Find the *horizontal range* R—that is, the horizontal distance from the starting point to where the ball hits the ground—and the ball's velocity just before it hits.



$$v_{0x} = v_0 \cos\alpha_0 = (37.0\text{ m/s})\cos 53.1° = 22.2\text{ m/s}$$

$$v_{0y} = v_0 \sin\alpha_0 = (37.0\text{ m/s})\sin 53.1° = 29.6\text{ m/s}$$

$$x = v_{0x}t = (22.2\text{ m/s})(2.00\text{ s}) = 44.4\text{ m}$$

$$y = v_{0y}t - \tfrac{1}{2}gt^2$$
$$= (29.6\text{ m/s})(2.00\text{ s}) - \tfrac{1}{2}(9.80\text{ m/s}^2)(2.00\text{ s})^2 = 39.6\text{ m}$$

$$v_x = v_{0x} = 22.2\text{ m/s}$$

$$v_y = v_{0y} - gt = 29.6\text{ m/s} - (9.80\text{ m/s}^2)(2.00\text{ s}) = 10.0\text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(22.2\text{ m/s})^2 + (10.0\text{ m/s})^2} = 24.4\text{ m/s}$$

$$\alpha = \arctan\left(\frac{10.0\text{ m/s}}{22.2\text{ m/s}}\right) = \arctan 0.450 = 24.2°$$

$$v_y = v_{0y} - gt_1 = 0$$

$$t_1 = \frac{v_{0y}}{g} = \frac{29.6\text{ m/s}}{9.80\text{ m/s}^2} = 3.02\text{ s}$$

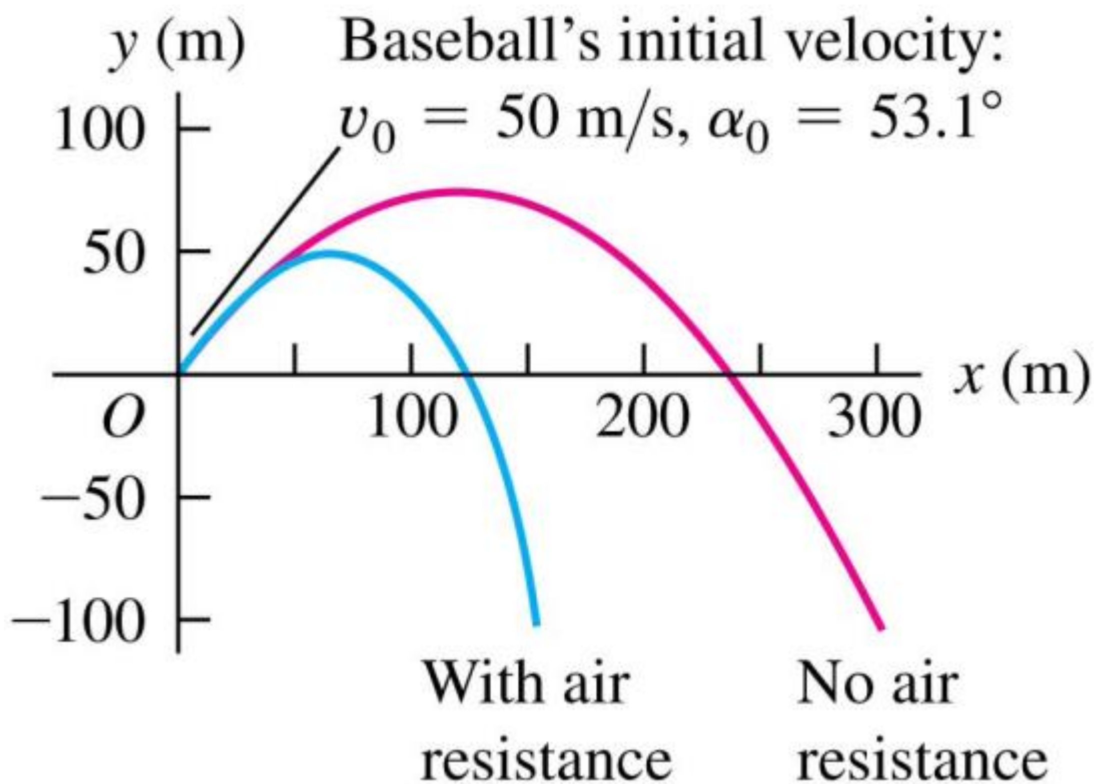
$$h = v_{0y}t_1 - \tfrac{1}{2}gt_1^2$$
$$= (29.6\text{ m/s})(3.02\text{ s}) - \tfrac{1}{2}(9.80\text{ m/s}^2)(3.02\text{ s})^2 = 44.7\text{ m}$$

$$R = v_{0x}t_2 = (22.2\text{ m/s})(6.04\text{ s}) = 134\text{ m}$$

$$v_y = v_{0y} - gt_2 = 29.6\text{ m/s} - (9.80\text{ m/s}^2)(6.04\text{ s})$$
$$= -29.6\text{ m/s}$$

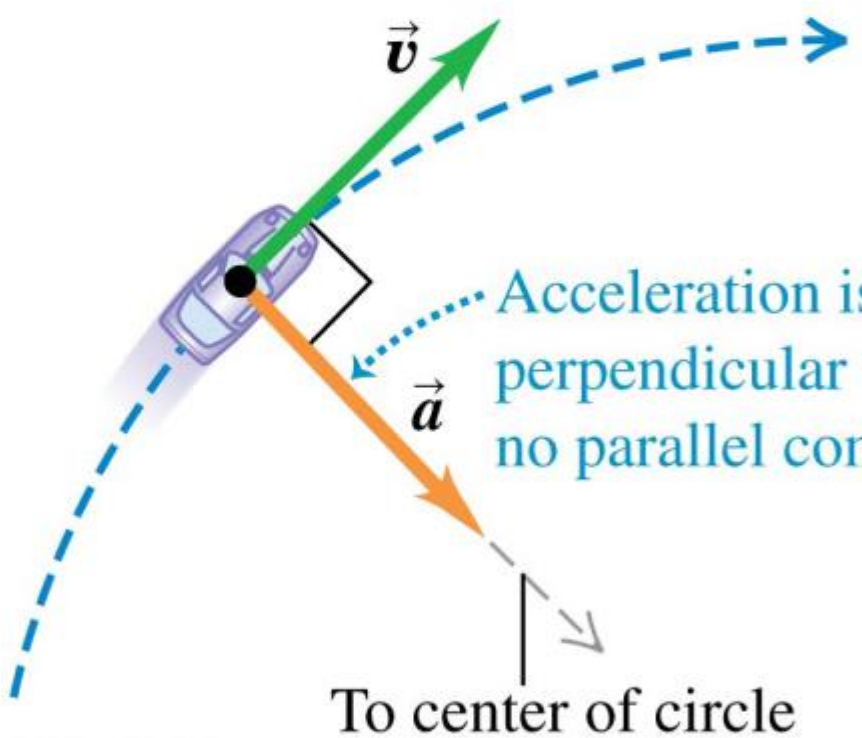
The effects of air resistance

- Calculations become more complicated.
- Acceleration is not constant.
- Effects can be very large.
- Maximum height and range decrease.
- Trajectory is no longer a parabola.



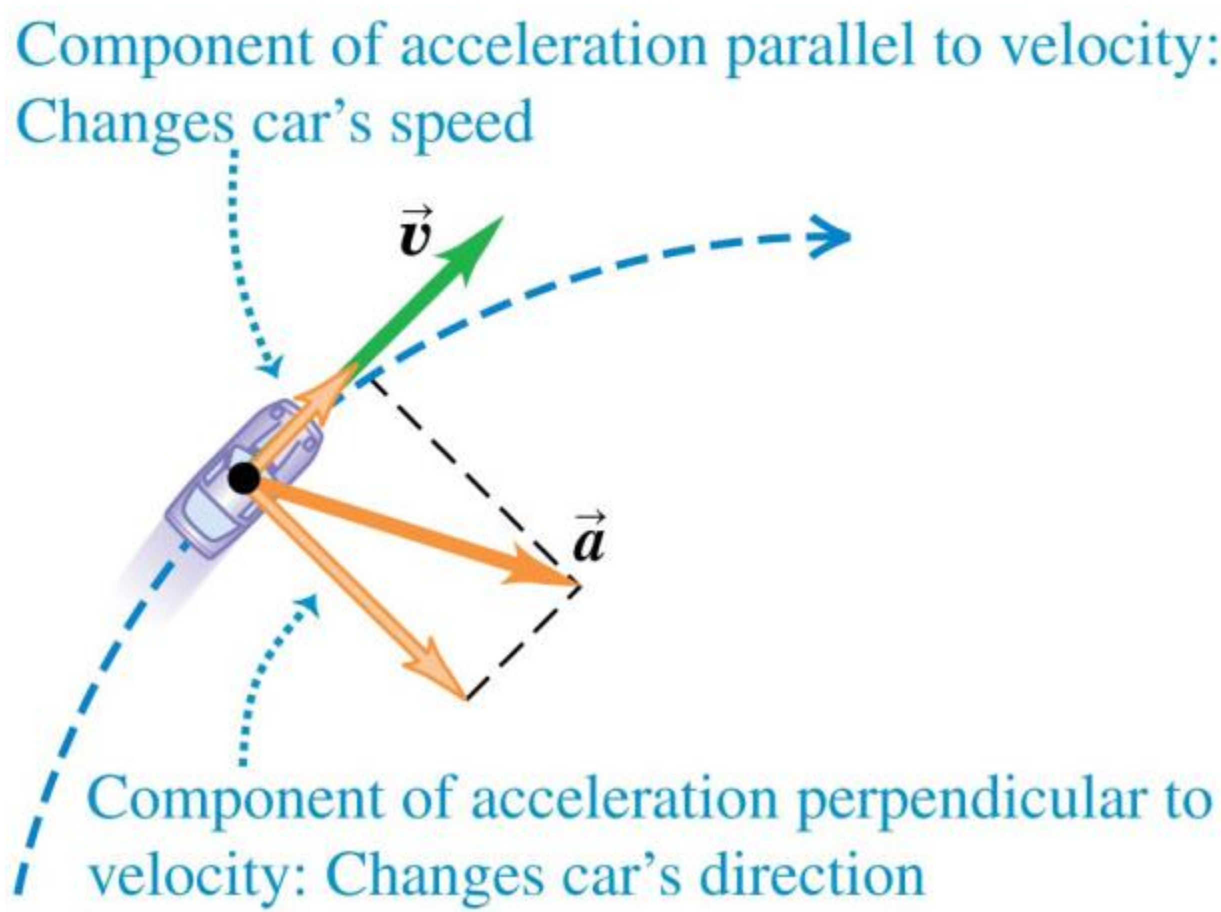
Motion in a circle

- **Uniform circular motion** is constant speed along a circular path.



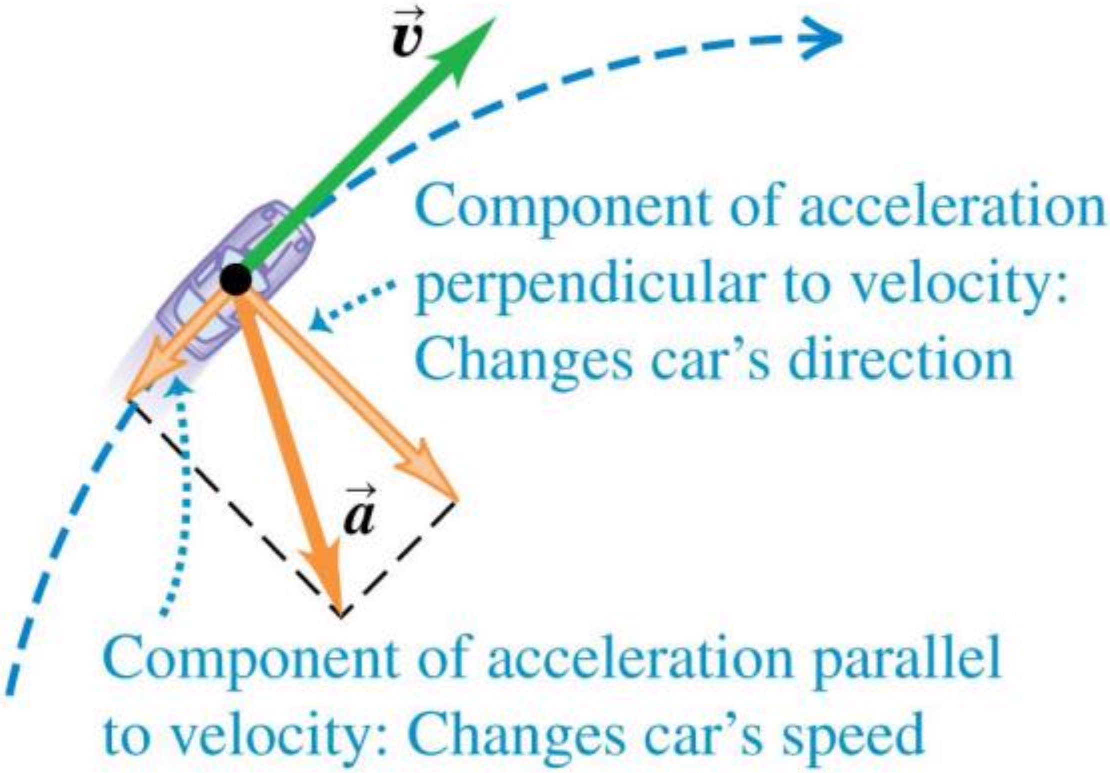
Motion in a circle

- Car speeding up along a circular path



Motion in a circle

- Car slowing down along a circular path



Acceleration for uniform circular motion

$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \qquad \text{or} \qquad |\Delta\vec{v}| = \frac{v_1}{R}\Delta s$$

$$a_{av} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$

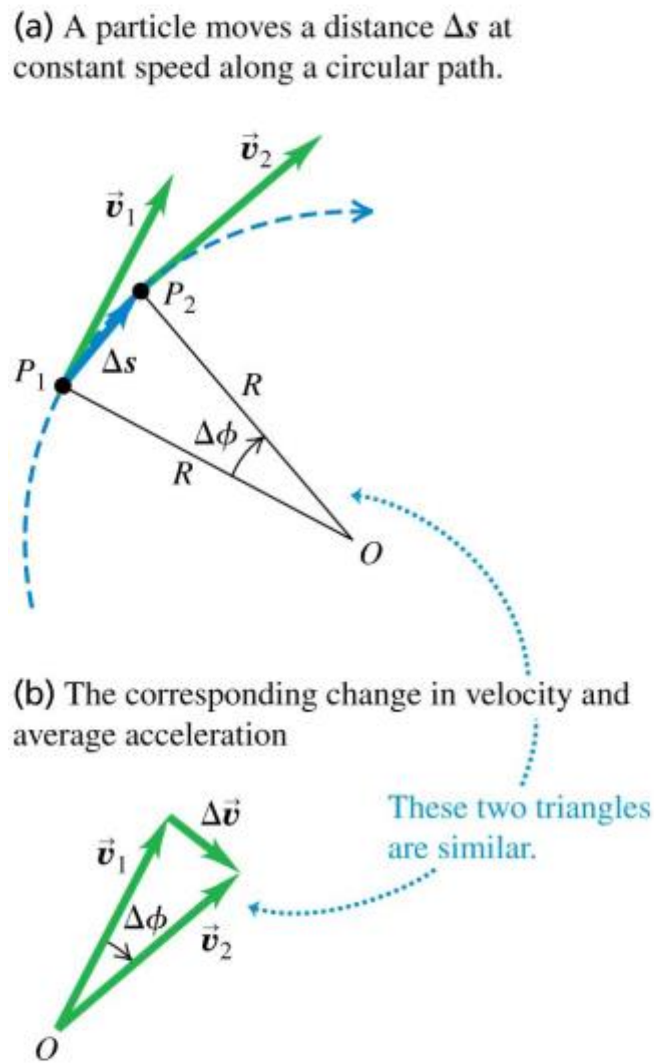
$$a = \lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$$

Magnitude of acceleration of an object in **uniform circular motion**

$$a_{rad} = \frac{v^2}{R}$$

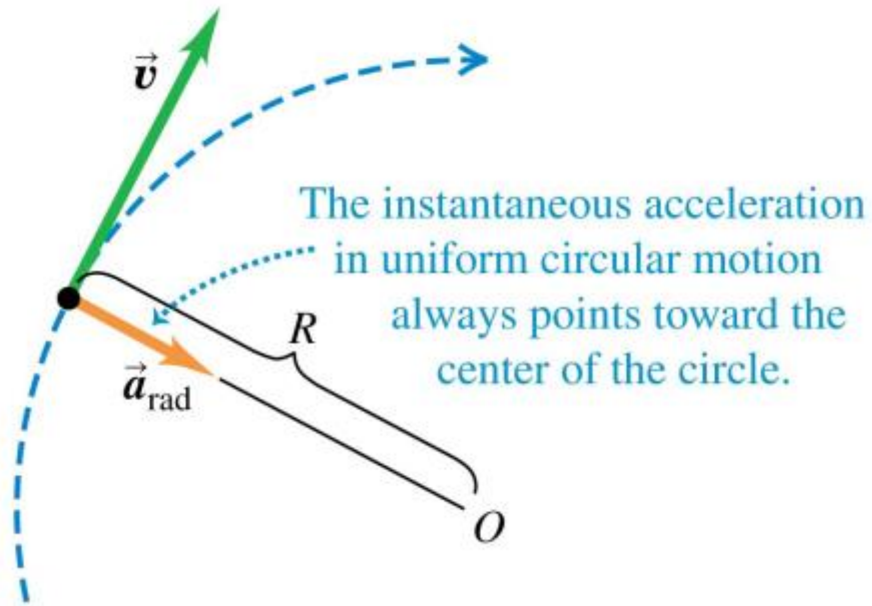
Speed of object

Radius of object's circular path



Acceleration for uniform circular motion

- For uniform circular motion, the instantaneous acceleration always points toward the center of the circle and is called the **centripetal acceleration**.



- The magnitude of the acceleration is $a_{rad} = v^2/R$.
- The *period* T is the time for one revolution, and $a_{rad} = 4\pi^2 R/T^2$.

EXAMPLE 3.11 CENTRIPETAL ACCELERATION ON A CURVED ROAD

An Aston Martin V8 Vantage sports car has a "lateral acceleration" of $0.96g = (0.96)(9.8\ \text{m/s}^2) = 9.4\ \text{m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h, or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$R = \frac{v^2}{a_{\text{rad}}} = \frac{(40\ \text{m/s})^2}{9.4\ \text{m/s}^2} = 170\ \text{m (about 560 ft)}$$

EXAMPLE 3.12 CENTRIPETAL ACCELERATION ON A CARNIVAL RIDE

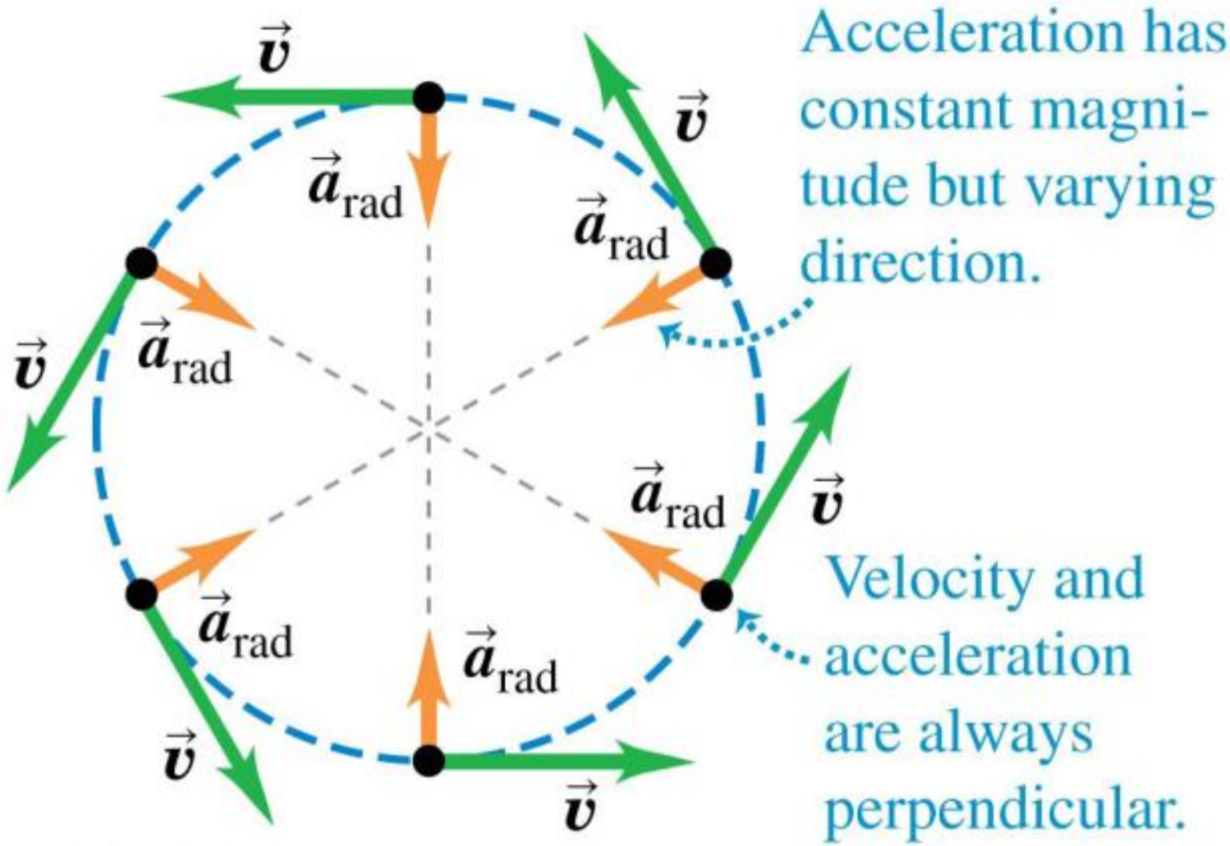
Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration?

$$v = \frac{2\pi R}{T} = \frac{2\pi(5.0\ \text{m})}{4.0\ \text{s}} = 7.9\ \text{m/s}$$

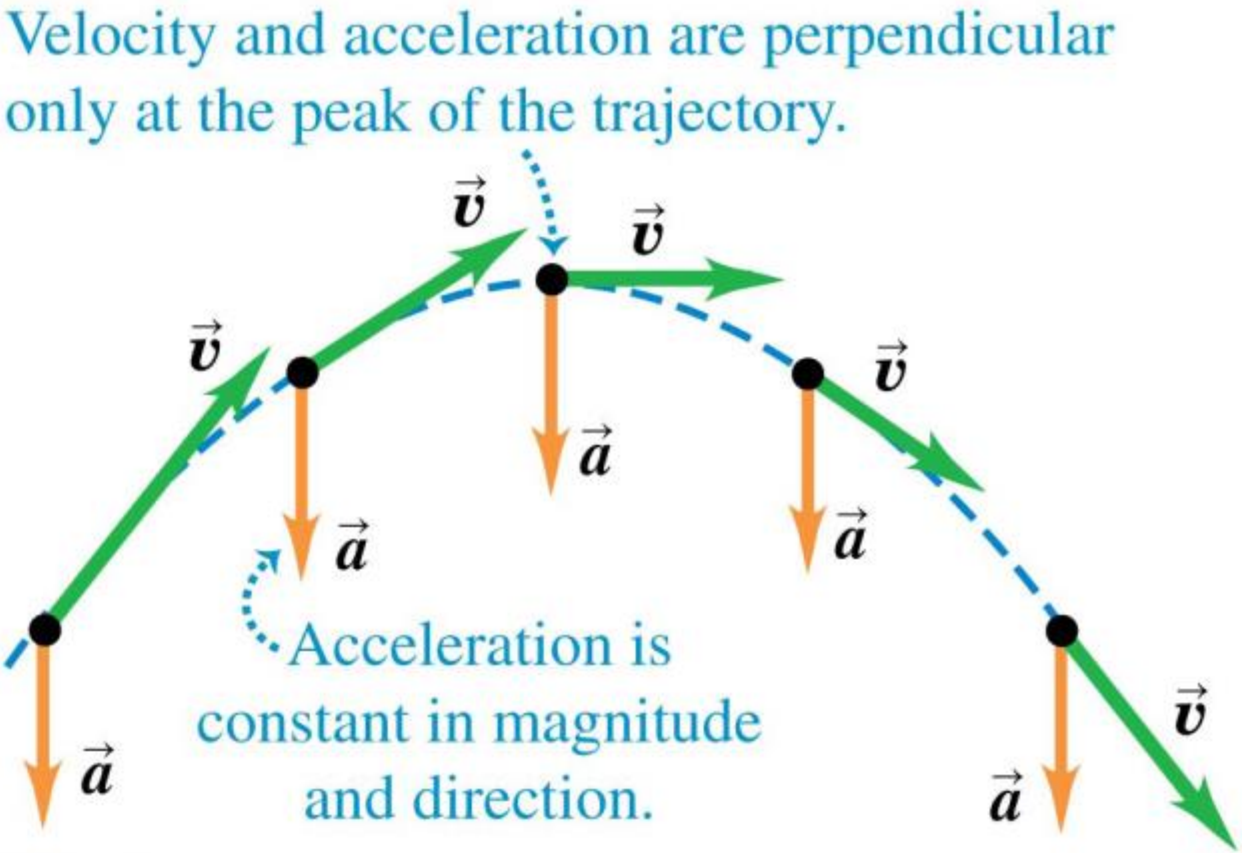
$$a_{\text{rad}} = \frac{4\pi^2(5.0\ \text{m})}{(4.0\ \text{s})^2} = 12\ \text{m/s}^2 = 1.3g$$

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{(7.9\ \text{m/s})^2}{5.0\ \text{m}} = 12\ \text{m/s}^2$$

Uniform circular motion

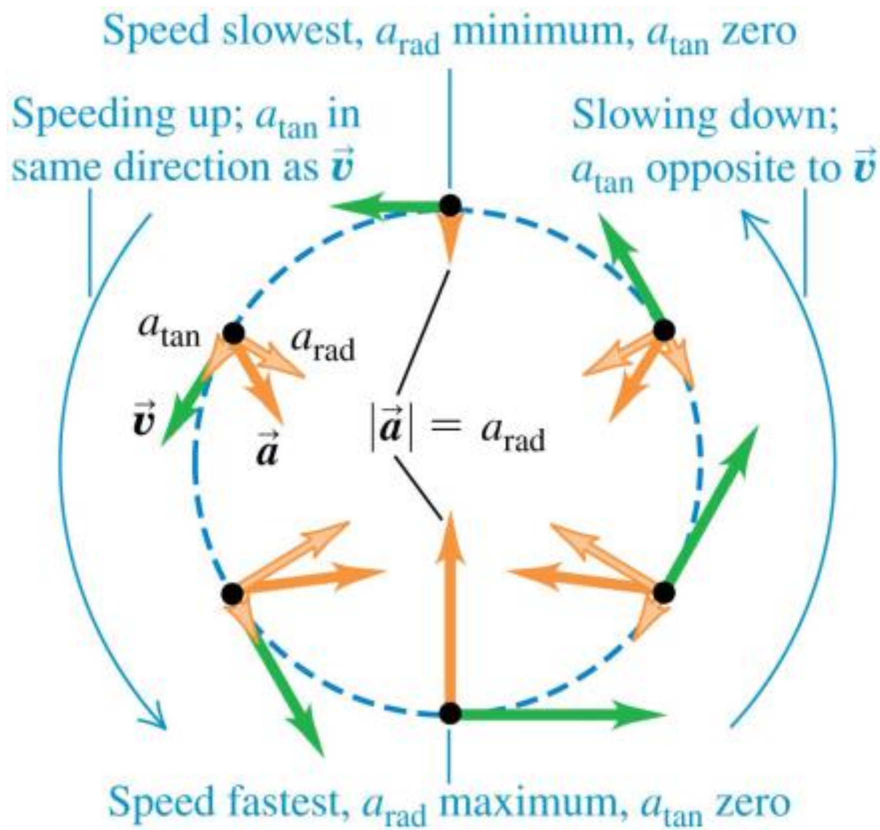


Projectile motion



Nonuniform circular motion

- If the speed varies, the motion is *nonuniform circular motion*.
- The radial acceleration component is still $a_{rad} = v^2/R$, but there is also a tangential acceleration component a_{tan} that is *parallel* to the instantaneous velocity.

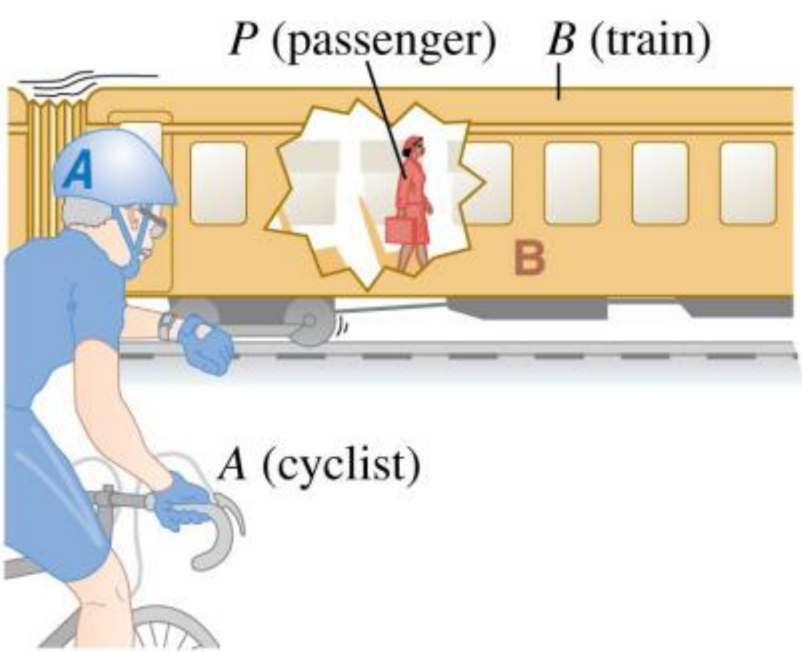


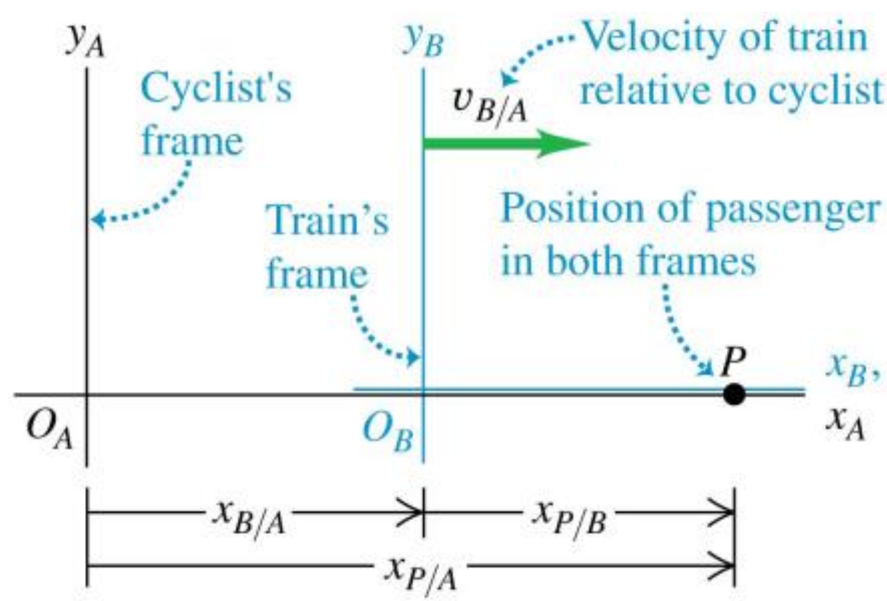
Relative velocity

- The velocity of a moving body seen by a particular observer is called the velocity *relative* to that observer, or simply the **relative velocity**.
- A **frame of reference** is a coordinate system plus a time scale.
- In many situations relative velocity is extremely important.

Relative velocity in one dimension

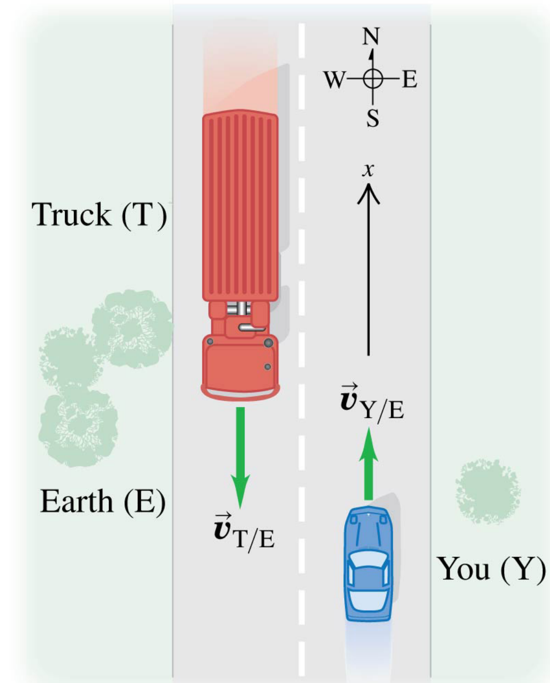
- If point P is moving relative to reference frame A, we denote the velocity of P relative to frame A as $v_{P/A}$.
- If P is moving relative to frame B and frame B is moving relative to frame A, then the x-velocity of P relative to frame A is $v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x}$.





EXAMPLE 3.13 RELATIVE VELOCITY ON A STRAIGHT ROAD

You drive north on a straight two-lane road at a constant 88 km/h. A truck in the other lane approaches you at a constant 104 km/h (**Fig. 3.33**). Find (a) the truck's velocity relative to you and (b) your velocity relative to the truck. (c) How do the relative velocities change after you and the truck pass each other? Treat this as a one-dimensional problem.



$$v_{T/E\text{-}x} = v_{T/Y\text{-}x} + v_{Y/E\text{-}x}$$

$$v_{T/Y\text{-}x} = v_{T/E\text{-}x} - v_{Y/E\text{-}x}$$

$$= -104\ \text{km/h} - 88\ \text{km/h} = -192\ \text{km/h}$$

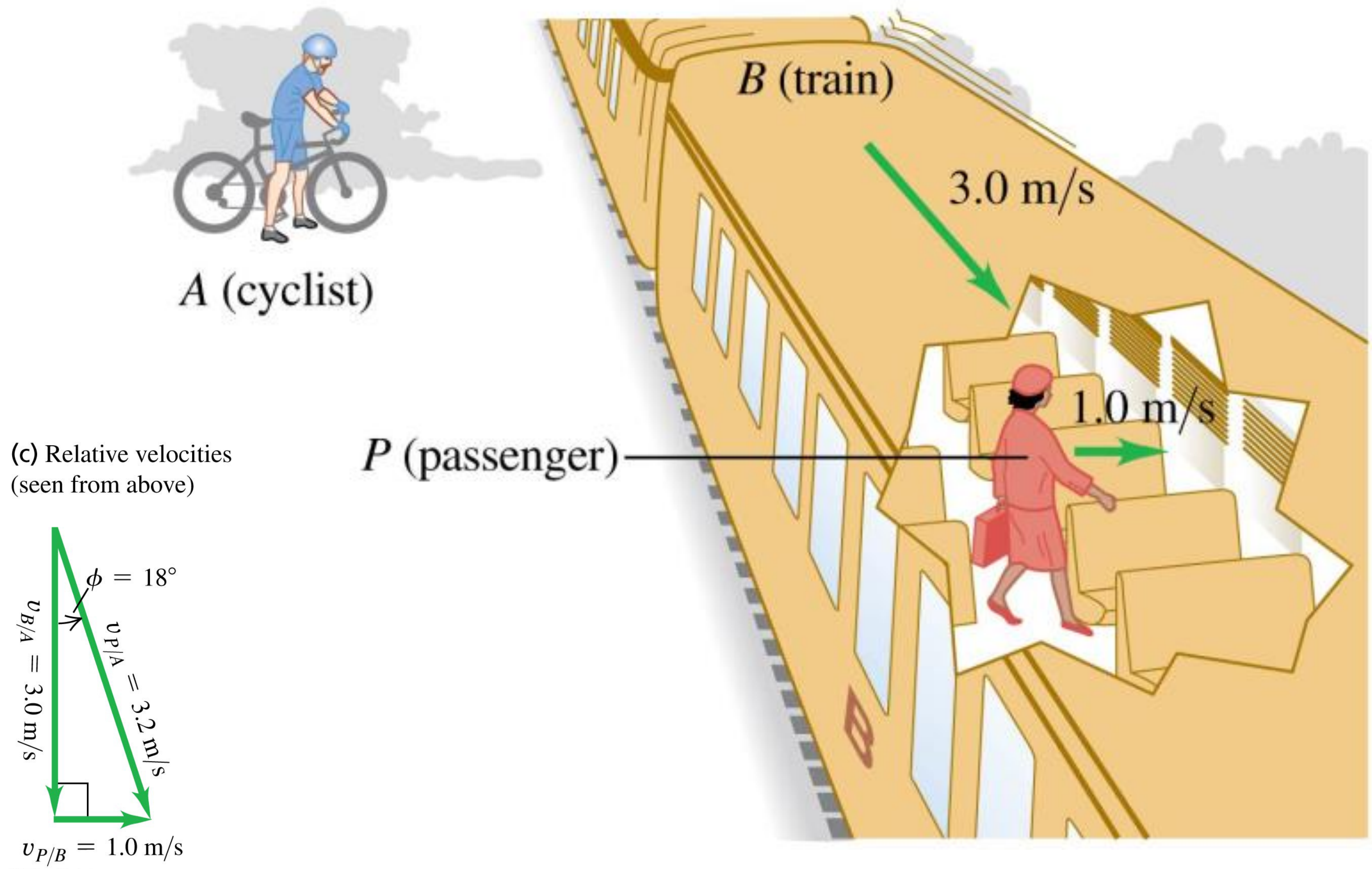
The truck is moving at 192 km/h in the negative x-direction (south) relative to you.

$$v_{Y/T\text{-}x} = -v_{T/Y\text{-}x} = -(-192\ \text{km/h}) = +192\ \text{km/h}$$

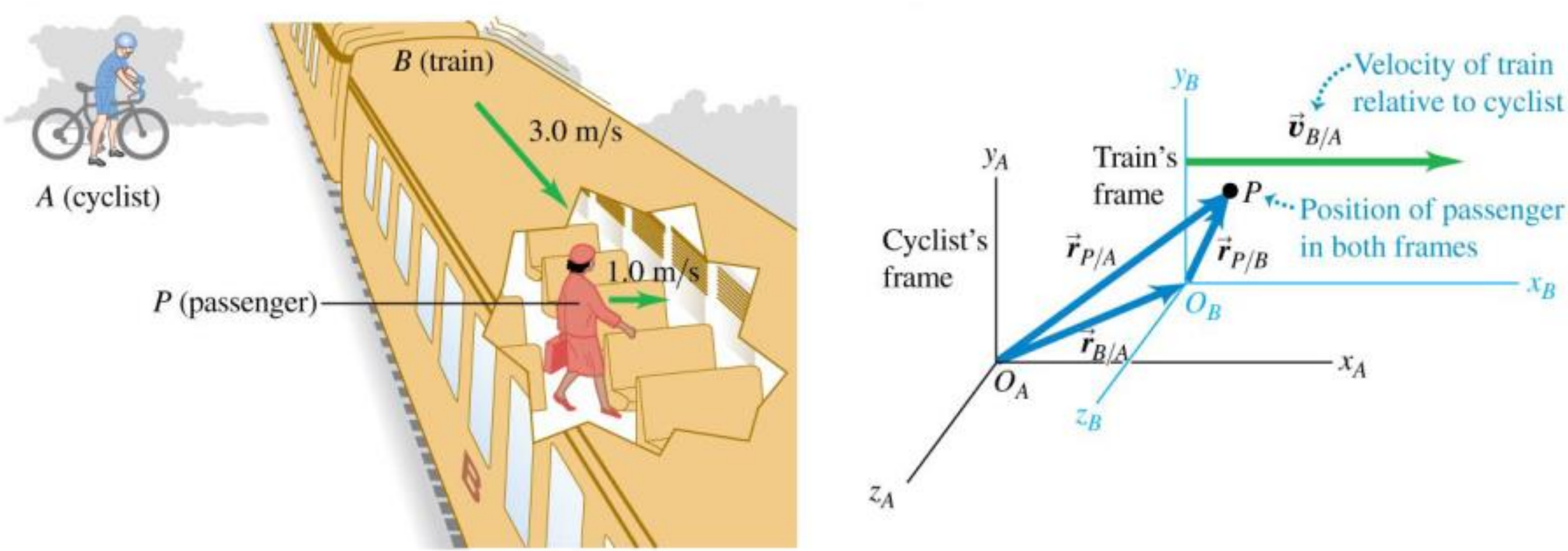
You are moving at 192 km/h in the positive x-direction (north) relative to the truck.

Relative velocity in two or three dimensions

- We extend relative velocity to two or three dimensions by using vector addition to combine velocities.



Relative velocity in two or three dimensions



Relative velocity in space:

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

Velocity of P relative to A

Velocity of P relative to B

Velocity of B relative to A

EXAMPLE 3.14 FLYING IN A CROSSWIND

An airplane's compass indicates that it is headed due north, and its airspeed indicator shows that it is moving through the air at 240 km/h. If there is a 100-km/h wind from west to east, what is the velocity of the airplane relative to the earth?

SOLUTION

IDENTIFY and SET UP: This problem involves velocities in two dimensions (northward and eastward), so it is a relative velocity problem using vectors. We are given the magnitude and direction of the velocity of the plane (P) relative to the air (A). We are also given the magnitude and direction of the wind velocity, which is the velocity of the air A with respect to the earth (E):

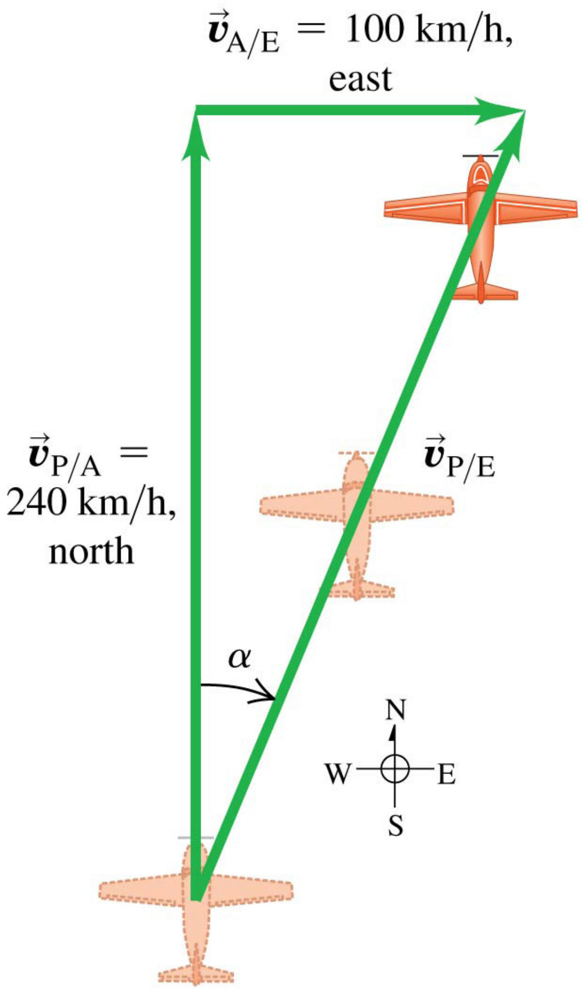
$$\vec{v}_{P/A} = 240\ \text{km/h} \qquad \text{due north}$$
$$\vec{v}_{A/E} = 100\ \text{km/h} \qquad \text{due east}$$

We'll use Eq. (3.35) to find our target variables: the magnitude and direction of velocity $\vec{v}_{P/E}$ of the plane relative to the earth.

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$$

$$v_{P/E} = \sqrt{(240\ \text{km/h})^2 + (100\ \text{km/h})^2} = 260\ \text{km/h}$$

$$\alpha = \arctan\left(\frac{100\ \text{km/h}}{240\ \text{km/h}}\right) = 23°\ \text{E of N}$$



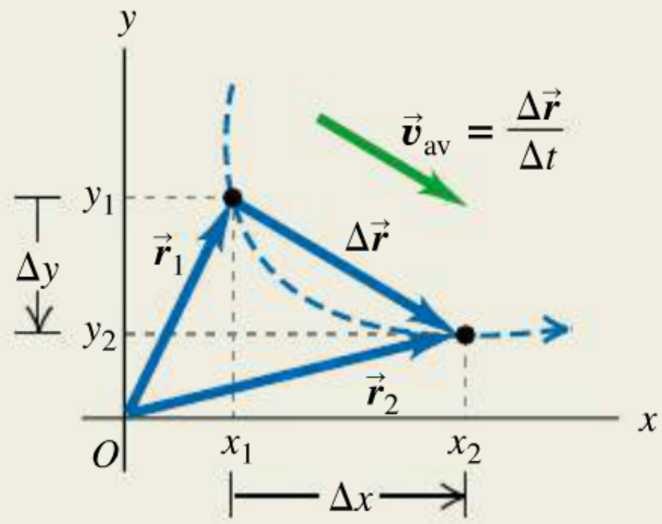
SUMMARY

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \tag{3.1}$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \tag{3.2}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \tag{3.3}$$

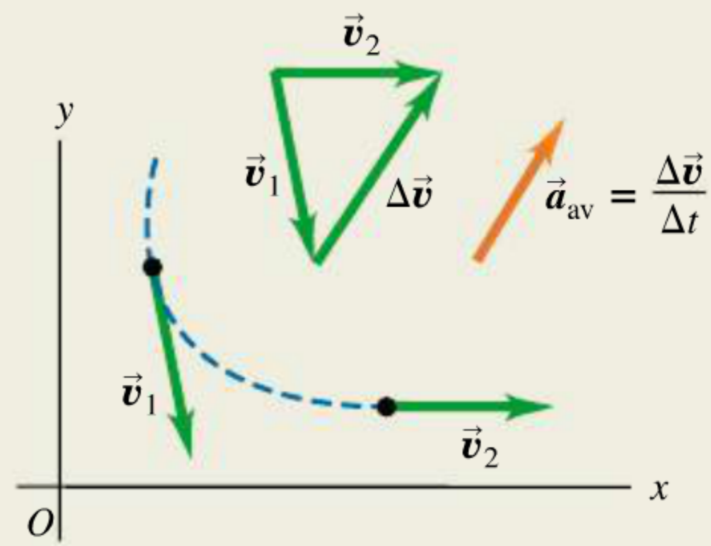
$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt} \tag{3.4}$$



$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \tag{3.8}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \tag{3.9}$$

$$a_x = \frac{dv_x}{dt}$$
$$a_y = \frac{dv_y}{dt} \tag{3.10}$$
$$a_z = \frac{dv_z}{dt}$$



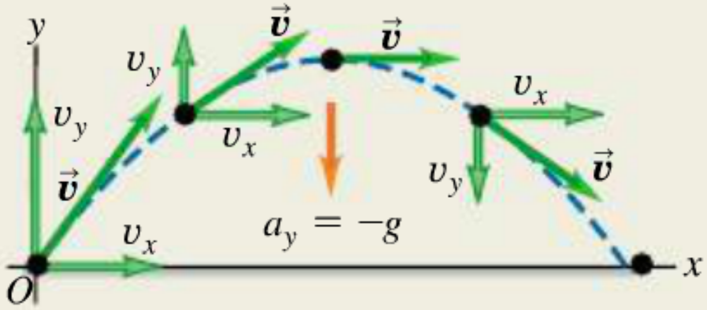
SUMMARY

$$x = (v_0 \cos \alpha_0)t \tag{3.19}$$

$$y = (v_0 \sin \alpha_0)t - \tfrac{1}{2}gt^2 \tag{3.20}$$

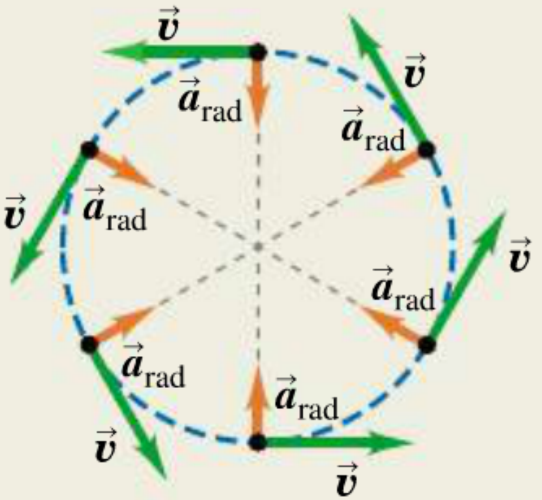
$$v_x = v_0 \cos \alpha_0 \tag{3.21}$$

$$v_y = v_0 \sin \alpha_0 - gt \tag{3.22}$$



$$a_{\text{rad}} = \frac{v^2}{R} \tag{3.27}$$

$$a_{\text{rad}} = \frac{4\pi^2 R}{T^2} \tag{3.29}$$



SUMMARY

$v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x}$ (3.32)
(relative velocity along a line)

$\vec{\boldsymbol{v}}_{P/A} = \vec{\boldsymbol{v}}_{P/B} + \vec{\boldsymbol{v}}_{B/A}$ (3.35)
(relative velocity in space)