1) Find the matrix elements of position and momentum operators on the Harmonic oscillator energy Eigenvalues basis. Show that the commutativity relation of [X,P].

2) Find the expectation value of the potential energy in the nth state of the harmonic oscillator.

3)

(a) Construct $\psi_2(x)$.

(b) Sketch ψ_0, ψ_1, and ψ_2.

(c) Check the orthogonality of ψ_0, ψ_1, and ψ_2, by explicit integration. *Hint:* If you exploit the even-ness and odd-ness of the functions, there is really only one integral left to do.

4) Find $\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, $\langle p^2\rangle$, and $\langle T\rangle$, for the nth stationary state of the harmonic oscillator, using the method of Example 2.5. Check that the uncertainty principle is satisfied.

5) A particle in the harmonic oscillator potential starts out in the state

$$\Psi(x,0) = A[3\psi_0(x) + 4\psi_1(x)].$$

(a) Find A.

(b) Construct $\Psi(x,t)$ and $|\Psi(x,t)|^2$.

6) (a) For a particle of mass m in a one-dimensional harmonic oscillator potential V = $m\omega^2x^2/2$, write down the most general solution to the **time-dependent Schrödinger** equation, $\psi(x,t)$, in terms of harmonic oscillator eigenstates $\phi_n(x)$.

(b) Using (a) show that the expectation value of x, (x), **as** a function of time can be written as ***A* cos *wt* + *B*** sin ***wt,*** where ***A*** and ***B*** are constants.

(c) Using (a) show explicitly that the time average of the potential energy satisfies $\langle V\rangle = \frac{1}{2}\langle E\rangle$ for a general $\psi(x,t)$.

Note the equality

$$\sqrt{\frac{m\omega}{\hbar}}\, x\phi_n = \sqrt{\frac{n+1}{2}}\,\phi_{n+1} + \sqrt{\frac{n}{2}}\,\phi_{n-1}\,.$$

7) At time $t = 0$ a particle in the potential $V(x) = m\omega^2 x^2/2$ is described by the wave function

$$\psi(x, 0) = A \sum_n (1/\sqrt{2})^n \psi_n(x),$$

where ψ_n (x) are eigenstates of the energy with eigenvalues $E_n = (n + 1/2)\hbar\omega$. You are given that $(\psi_n, \psi_{n'}) = \delta_{nn'}$.

(a) Find the normalization constant A.
(b) Write an expression for $\psi(x,t)$ for $t > 0$.
(c) Show that $|\psi(x,t)|^2$ is a periodic function of time and indicate the longest period τ.
(d) Find the expectation value of the energy at $t = 0$.

(Berkeley)

8) The wave function of the ground state of a harmonic oscillator of force constant k and mass m is

$$\psi_0(x) = (\alpha/\pi)^{1/4} e^{-\alpha x^2/2}, \quad \alpha = m\omega_0/\hbar, \quad \omega_0^2 = k/m.$$

Obtain an expression for the probability of finding the particle outside the classical region.

(Wisconsin)

9) Prove the following results for an eigenstate of the Hamiltonian of a one-dimensional harmonic oscillator.

(a) The expectation values of the position and momentum are zero,
(b) The expectation values of the potential and kinetic energies are equal.
(c) Δ_x and Δ_p, the uncertainties in position and momentum, satisfy the relation $\Delta_x \Delta_p = (n + \frac{1}{2})\hbar$, where n is the quantum number of the state.
(d) Show that the ground-state eigenfunction u_0 is a Gaussian, and hence that the parity of the state n is even for even n, and odd for odd n.

10) Consider a one-dimensional harmonic oscillator with potential energy $V(x) = \frac{1}{2}m\omega^2 x^2$. The initial ($t = 0$) wave function of the system is

$$\psi(x, 0) = \frac{1}{\sqrt{2}}\left(1 - \frac{x}{|x|}\right) f(x)$$

where $f(x)$ is a real ($f^*(x) = f(x)$) normalized function that is odd under space reflection $x \to -x$, i.e. $f(-x) = -f(x)$.

(a) Is $\psi(x, 0)$ normalized?
(b) What is the initial probability density at the point $x = 0$?
(c) What is the initial probability of finding the particle in the region $[0, +\infty]$? What is the initial probability for the region $[-\infty, 0]$?

11) A particle of mass m and electric charge q can move only in one dimension and is subject to a harmonic force and a homogeneous electrostatic field. The Hamiltonian operator for the system is

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2 - q\mathcal{E}x$$

(a) Solve the energy eigenvalue problem.
(b) If the system is initially in the ground state of the unperturbed harmonic oscillator, $|\psi(0)\rangle = |0\rangle$, what is the probability of finding it in the ground state of the full Hamiltonian?
(c) Assume again that the system is initially in the unperturbed harmonic oscillator ground state and calculate the probability of finding it in this state again at a later time.
(d) With the same initial condition calculate the probability of finding the particle at a later time in the first excited state of the unperturbed harmonic oscillator.
(e) Consider the *electric dipole moment* $d \equiv qx$ and calculate its vacuum expectation value in the evolved state $|\psi(t)\rangle$, assuming again that we start from the unperturbed vacuum state at $t = 0$.