1) **Solve the system of linear equations**

a)
$$y + x = 0$$
$$2x + y = 0$$

Consider the given system of equations as represented below.

$$y + x = 0 \quad (1)$$
$$2x + y = 0 \quad (2)$$

Now subtracting equation (1) from equation (2) we get as follows.

$$(2x + y = 0) - (y + x = 0)$$

$$\Rightarrow (2x + y - y - x) = (0 - 0)$$
$$\Rightarrow x + 0y = 0$$
$$\Rightarrow x = 0 \quad (3)$$

Now substituting (3) in equation (1) and solving for y as represented below.

$$y + x = 0$$
$$\Rightarrow y + (0) = 0$$
$$\Rightarrow y = 0$$

The solution of the given system of equations is $\boxed{x = 0,\ y = 0}$.

b)
$$x + y = -1$$
$$3x + 2y = 0$$

Consider the given system of equations as represented below.

$$x + y = -1 \quad (1)$$
$$3x + 2y = 0 \quad (2)$$

Now now multiplying equation (1) with 2 and subtracting it from equation (2) we get as follows.

$$(3x + 2y = 0) - 2 \times (x + y = -1)$$

$$\Rightarrow (3x - 2x) + (2y - 2y) = (0 + 2)$$
$$\Rightarrow x + 0y = 2$$
$$\Rightarrow x = 2 \quad (3)$$

Now substituting (3) in equation (1) and solving for y as represented below.

$$x + y = -1$$
$$\Rightarrow (2) + y = -1$$
$$\Rightarrow y = -1 - 2$$
$$\Rightarrow y = -3$$

The solution of the given system of equations is $\boxed{x = 2,\ y = \text{-}3}$.

2) **Solve the system using either Gaussian elimination with back-substitution or Gauss-Jordan elimination.**

$$\begin{aligned} x_1 \qquad\quad - 3x_3 &= -2 \\ 3x_1 + x_2 - 2x_3 &= 5 \\ 2x_1 + 2x_2 + x_3 &= 4 \end{aligned}$$

$$\begin{bmatrix}1 & 0 & -3 & -2\\ 3 & 1 & -2 & 5\\ 2 & 2 & 1 & 4\end{bmatrix} \quad -3R_1 + R_2 \to R_2 \quad \begin{bmatrix}1 & 0 & -3 & -2\\ 0 & 1 & 7 & 11\\ 2 & 2 & 1 & 4\end{bmatrix}$$

$$-2R_1 + R_3 \to R_3 \quad \begin{bmatrix}1 & 0 & -3 & -2\\ 0 & 1 & 7 & 11\\ 0 & 2 & 7 & 8\end{bmatrix}$$

$$-2R_2 + R_3 \to R_3 \quad \begin{bmatrix}1 & 0 & -3 & -2\\ 0 & 1 & 7 & 11\\ 0 & 0 & -7 & -14\end{bmatrix}$$

$$-\frac{1}{7}R_3 \to R_3 \quad \begin{bmatrix}1 & 0 & -3 & -2\\ 0 & 1 & 7 & 11\\ 0 & 0 & 1 & 2\end{bmatrix}$$

$$-7R_3 + R_2 \to R_2 \quad \begin{bmatrix}1 & 0 & -3 & -2\\ 0 & 1 & 0 & -3\\ 0 & 0 & 1 & 2\end{bmatrix}$$

$$3R_3 + R_1 \to R_1 \quad \begin{bmatrix}1 & 0 & 0 & 4\\ 0 & 1 & 0 & -3\\ 0 & 0 & 1 & 2\end{bmatrix}$$

The associated equivalent system is:

$$x_1 = 4 \qquad (1)$$
$$x_2 = -3 \qquad (2)$$
$$x_3 = 2 \qquad (3)$$

$$x_1 = 4 \quad x_2 = -3 \quad x_3 = 2$$

$$A = \begin{bmatrix}3 & 2 & -1\\ 2 & 4 & 5\\ 0 & 1 & 2\end{bmatrix}, \quad B = \begin{bmatrix}0 & 2 & 1\\ 5 & 4 & 2\\ 2 & 1 & 0\end{bmatrix}$$

3) Find 2*A*–*B*

$$2A - B = 2A + (-B)$$

$$\begin{bmatrix}6-0 & 4-2 & -2-1\\ 4-5 & 8-4 & 10-2\\ 0-2 & 2-1 & 4-0\end{bmatrix} =$$

$$\begin{bmatrix}6 & 2 & -3\\ -1 & 4 & 8\\ -2 & 1 & 4\end{bmatrix}$$

4) Find AB

$$A = \begin{bmatrix}2 & -2\\ -1 & 4\end{bmatrix}, \quad B = \begin{bmatrix}4 & 1\\ 2 & -2\end{bmatrix}$$

and

$$A = \begin{bmatrix}3 & 2 & 1\\ -3 & 0 & 4\\ 4 & -2 & -4\end{bmatrix}, \quad B = \begin{bmatrix}1 & 2\\ 2 & -1\\ 1 & -2\end{bmatrix}$$

(a) $AB =$

$$\begin{bmatrix}(2)(4)+(-2)(2) & (2)(1)+(-2)(-2)\\ (-1)(4)+(4)(2) & (-1)(1)+(4)(-2)\end{bmatrix} =$$

$$\begin{bmatrix}8+-4 & 2+4\\ -4+8 & -1+-8\end{bmatrix} =$$

$$\begin{bmatrix}4 & 6\\ 4 & -9\end{bmatrix}$$

$$A \cdot B = \begin{bmatrix}3 & 2 & 1\\ -3 & 0 & 4\\ 4 & -2 & 4\end{bmatrix} \cdot \begin{bmatrix}1 & 2\\ 2 & -1\\ 1 & -2\end{bmatrix}$$

$$= \begin{bmatrix}3+4+1 & 6-2-2\\ -3+0+4 & -6+0-8\\ 4-4-4 & 8+2+8\end{bmatrix}$$

$$= \begin{bmatrix}8 & 2\\ 1 & -14\\ -4 & 18\end{bmatrix}$$

5) **Find the A.** $(2A)^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Inverse of 2×2 matrix formula

Suppose we are given some 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then, its **inverse** is given with following formula:

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix},$$

where $\underbrace{\det A = ad - bc}_{\text{determinant of } A}$.

$(cA^{-1})^{-1} = cA$

This holds for any c.

$$[(2A)^{-1}]^{-1} = \frac{1}{[1 \cdot 4 - 2 \cdot 3]} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2} \cdot \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$\Rightarrow$

$$A = -\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$$

Simply use formula from above and follow the calculations.

$$A = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$$

6) $\begin{vmatrix} 1 & 7 & -3 \\ 1 & 3 & 1 \\ 4 & 8 & 1 \end{vmatrix}$ **Use elementary row or column operations to find the determinant. ,**

Using Theorem 3.3, modify given matrix to obtain triangular matrix.

$\begin{vmatrix} 1 & 7 & -3 \\ 1 & 3 & 1 \\ 4 & 8 & 1 \end{vmatrix}$, add -1 times first row to second and add -4 times first row to third

$\sim \begin{vmatrix} 1 & 7 & -3 \\ 0 & -4 & 4 \\ 0 & -20 & 13 \end{vmatrix}$, add -5 times second row to thrid

$\sim \begin{vmatrix} 1 & 7 & -3 \\ 0 & -4 & 4 \\ 0 & 0 & -7 \end{vmatrix}$

Matrix is triangular meaning that its determinant can be evaluated as product of diagonal elements.

$\det = 1 \cdot (-4) \cdot (-7) = 28$

7) Show that $|A| = |A^T|$ for the matrix below.

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 0 & 0 \\ -4 & -1 & 5 \end{bmatrix}$$

To find the determinant of A, expand by cofactors in the second *row* to obtain

$$|A| = 2(-1)^3 \begin{vmatrix} 1 & -2 \\ -1 & 5 \end{vmatrix}$$
$$= (2)(-1)(3)$$
$$= -6.$$

To find the determinant of

$$A^T = \begin{bmatrix} 3 & 2 & -4 \\ 1 & 0 & -1 \\ -2 & 0 & 5 \end{bmatrix}$$

expand by cofactors in the second *column* to obtain

$$|A^T| = 2(-1)^3 \begin{vmatrix} 1 & -1 \\ -2 & 5 \end{vmatrix}$$
$$= (2)(-1)(3)$$
$$= -6.$$

8) **Use a determinant of the coefficient matrix to determine whether the system of linear equations has a unique solution**

$$\begin{aligned} x_1 - 3x_2 &= 2 \\ 2x_1 + x_2 &= 1 \end{aligned}$$

Coefficient matrix is,

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}$$

Now $|A| = 1 \times 1 + 3 \times 2 = 7 \neq 0$

Hence given system of linear equations has unique solutions.

$$\begin{aligned} x_1 + x_2 - x_3 &= 4 \\ 2x_1 - x_2 + x_3 &= 6 \\ 3x_1 - 2x_2 + 2x_3 &= 0 \end{aligned}$$

The coefficient matrix corresponding to the given system of linear equation is

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ 3 & -2 & 2 \end{bmatrix}$$

By elementary column transformation we have :

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\Big\downarrow C_2 + C_3$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 0 \\ 3 & -2 & 0 \end{bmatrix}$$

Since the third column of the matrix is zero, by evaluating determinant of A w.r.t the third column gives $|A| = 0$.

Thus, $\det(A) = |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ 3 & -2 & 2 \end{vmatrix} = 0.$

Hence the given system of linear equation doesnot have unique solution.

The coefficient matrix corresponding to the given system of linear equation

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ 3 & -2 & 2 \end{bmatrix}$$

is singular, hence the system of linear equation doesnot have unique solution.